

AD-A218 079

DTIC FILE COPY

(2)

NPS55-89-10

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DTIC
ELECTED
FEB 20 1990
S D



A SATELLITE SIMULATION PROGRAM

REX H. SHUDDER

SEPTEMBER 1989

Approved for Public Release; Distribution Unlimited

Prepared for:

Naval War College
Wargaming Department
Newport, RI 02841-5010

90 02 16 004

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Ralph W. West, Jr.
Superintendent

Harrison Shull
Provost

This work was supported by the Naval War College and was funded by the Naval Post-graduate School.

Reproduction of all or part of this report is authorized.

This report was prepared by:



REX H. SHUDDE, Associate Professor
Department of Operations Research

Reviewed by:



PETER PURDUE, Chairman
Department of Operations Research

Released by:



KNEALE T. MARSHALL
Acting Dean of Information and Policy

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS			
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.			
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NPS55-89-10		5. MONITORING ORGANIZATION REPORT NUMBER(S)			
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School	6b. OFFICE SYMBOL (If applicable) Code 55	7a. NAME OF MONITORING ORGANIZATION Naval War College, Wargaming Department			
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		7b. ADDRESS (City, State, and ZIP Code) Newport, RI 02841-5010			
8a. NAME OF FUNDING SPONSORING ORGANIZATION Naval Postgraduate School	8b. OFFICE SYMBOL (If applicable) 55Su	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER O&MN, Direct Funding			
8c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943		10. SOURCE OF FUNDING NUMBERS			
		PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) A SATELLITE SIMULATION PROGRAM					
12. PERSONAL AUTHOR(S); Rex H. Shudde					
13a. TYPE OF REPORT Technical Report	13b. TIME COVERED FROM _____ TO _____	14. DATE OF REPORT (Year, Month, Day)		15. PAGE COUNT 51	
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Satellite, Satellite Simulation, Satellite Vulnerability, Satellite Tracking, Satellite Orbit, Satellite Culmination, Ground Track			
19. ABSTRACT (Continue on reverse if necessary and identify by block number) This report presents a satellite simulation algorithm written for the IBM PC and clones in Microsoft QuickBASIC 4.5. The program can either generate an ephemeris of satellite ground-track position and location with respect to a fixed ground station, or determine the position and time the satellite is above the horizon of a specified ground station. Provision is made for the program to handle multiple satellites and multiple ground stations. Input is either by way of the computer keyboard or by separate files for satellites and ground stations. Output is written both on the screen and to an ASCII output file which can be accessed by a data based management program.					
20. DISTRIBUTION AVAILABILITY OF ABSTRACT <input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED			
22a. NAME OF RESPONSIBLE INDIVIDUAL Rex H. Shudde		22b. TELEPHONE (Include Area Code) (408) 646-2203		22c. OFFICE SYMBOL Code 55Su	

A SATELLITE SIMULATION PROGRAM

by

Rex H. Shudde
NAVAL POSTGRADUATE SCHOOL
Monterey, California

September 1989

DISCLAIMER

The reader is cautioned that the computer program presented in this report may not have been exercised for all cases of interest. While every effort has been made to ensure that the program is free from computational and logic errors, it cannot be considered validated. Any application of this program without additional verification is at the risk of the user.

Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution / _____	
Availability Codes	
Dist	Avail and/or Special
A-1	



ABSTRACT

This report presents a satellite simulation algorithm written for the IBM PC and clones in Microsoft QuickBASIC 4.5. The program can either generate an ephemeris of satellite ground-track position and location with respect to a fixed ground station, or determine the position and time the satellite is above the horizon of a specified ground station. Provision is made for the program to handle multiple satellites and multiple ground stations. Input is either by way of the computer keyboard or by separate files for satellites and ground stations. Output is written both on the screen and to an ASCII output file which can be accessed by a data based management program.

CONTENTS

I. INTRODUCTION	1
II. OPERATING INSTRUCTIONS	
A. Overview	1
B. Input	2
C. Output	5
III. SOURCES	9
IV. DISCUSSION	10
APPENDICIES	
A. The Simplfied Kinematics Algorithm	11
B. The Time of Culmination, Rise and Set Algorithm	13
C. The Satellite Heading Computation	19
D. The SATSTA Program Listing	21
E. The High Accuracy Kinematics Routine	43
REFERENCES	49
DISTRIBUTION	51

Intentionally blank.

I. INTRODUCTION

This report presents a satellite simulation algorithm written for the IBM PC and clones in Microsoft QuickBASIC 4.5. The program can either generate an ephemeris of satellite ground-track position and location with respect to a fixed ground station, or determine the position and time the satellite is above the horizon of a specified ground station. Provision is made for the program to handle multiple satellites and multiple ground stations. Input is either by way of the computer keyboard or by separate files for satellites and ground stations. Output is written both on the screen and to an ASCII output file which can be accessed by a data based management program.

Two versions of the satellite kinematics subroutine pos.update are presented. These algorithms were provided by NAVSPASUR [Ref. 1]. The first is a very accurate algorithm based upon a paper by Brouwer [Ref. 2] and is contained in NAVSPASUR's FORTRAN program called SHOWALL. A listing of the BASIC version of this algorithm is presented in Appendix E. The second algorithm is a much simplified approximation to the Brouwer model. Equations for the simplified model are contained in Appendix A, and the BASIC implementation is embedded in the listing of the full simulation program in Appendix D.

The next sections contain program operating instructions, structure of the input/output files, and suggestions for future improvements. The computation of the times of culmination, rise and set are in Appendix B, and the computation of satellite heading is in Appendix C. Readers unfamiliar with the nomenclature of satellite orbital elements should consult any standard work such as Escobal [Ref. 3, Ch. 3].

II. OPERATING INSTRUCTIONS

A. Overview. The source code is named **SATSTA.BAS** and the executable code is named **SATSTA.EXE**. To run, enter SATSTA at the DOS prompt and press the ↲ key.

The first menu (Fig. 1) requests a data input mode. Pressing 1 or 2 will require the data for one satellite and one ground station to be input from the keyboard. Pressing 3 or 4 assumes that two data files are accessible: either ELEMENTS.DAT or ELEMENTS.NSS containing the orbital elements for one or more satellites, and STATION.DAT containing the location of one or more ground stations—the structure of these files is discussed in the next section. No matter which of these four options is chosen, a simulation starting time, ending time and time-step increment will be required input from the keyboard in the third menu.

The second menu (Fig. 2) requests a run mode. Option 1 creates an ephemeris of satellite geographic position and satellite position with respect to the ground station for equally spaced times for the entire simulation duration. Option 2 computes the satellite

geographic position and satellite position with respect to the ground station only for those times during the simulation duration at which the satellite is above the horizon of the ground station.

The third menu (Fig. 3) requests the simulation-times, consisting of a starting time, an ending time, and a time-step increment (Δt).

The fourth and fifth menus (Figs. 4 and 5, respectively) appear only if the first menu (data input mode) options 1 or 2 were chosen. Menu four requests the satellite epoch and elements, and menu five requests the station coordinates.

No matter which options are chosen, output is directed to the screen and is appended to file **OUTPUT.DAT**. If the file does not exist, it is created. This ASCII file is structured so that it can be used as input to a data base management program, if desired. The structure of this file is discussed in a later section.

B. Input. Menu 1 requests the selection of a data input mode. Depending upon the source of satellite elements, there are two ways in which these data can be input. The first is using standard "epoch of date" elements in which the time of perigee passage is the epoch. The second is the NAVSPASUR "One-Line Charlie" elements in which the longitude (or right ascension) of the ascending node has been referenced to 0000 hours, 1 January 1985. The "One-Line Charlie" option has been added for those users who may obtain satellite elements from NAVSPASUR. An example of "epoch of date" input is shown in Fig. 3.

```
Select data input mode 1, 2, 3 or 4:  
Keyboard input for satellite & ground station:  
    1. Epoch of date.  
    2. NAVSPASUR Reference Date.  
Data file input for satellite & ground station:  
    3. Epoch of date.  
    4. NAVSPASUR One-Line Charlie.
```

Figure 1. Menu 1—Select data input mode.

If data file input Option 3 is chosen from Menu 1, two ASCII data files, **ELEMENTS.DAT** and **STATION.DAT**, must have been previously created. The **ELEMENTS.DAT** file must contain a single line of for each satellite of interest. Each line must contain 10 entries, each enclosed in double quotations and separated by a blank space (Fig. 6). The entries are: (1) a satellite identification consisting of at most five alphanumeric characters; (2) the epoch date (date of perigee passage) in the form **yyyy.mmdd**, where **yyyy** is the year, **mm** is the month, and **dd** is the day; (3) the epoch time (time of perigee passage) in the form **hh.mmss**, where **hh** is the hour, **mm** is the minutes, and **ss** is the seconds; (4) the orbital eccentricity; (5) the

longitude (or right ascension) of the ascending node in degrees; (6) the orbital inclination in degrees; (7) the argument of perigee in degrees; (8) the mean anomaly in degrees; (9) the mean motion in revolutions per day; and (10) the orbital decay in revolutions per day-squared.

If data file input Option 4 is chosen from Menu 1, two ASCII data files, **ELEMENTS.NSS** and **STATION.DAT**, must have been previously created. The **ELEMENTS.NSS** file must contain a single line of for each satellite of interest in the NAVSPASUR format. Each line must contain 10 entries starting in column 1 and ending in column 65 (Fig. 7). The entries are: (1) a satellite identification consisting of at most five alphanumeric characters (columns 1-5); (2) the mean anomaly in revolutions (columns 6-13); (3) the mean motion in radians per Herg¹ (columns 14-21); (4) the orbital decay in radians per Herg-squared (columns 22-27); (5) the orbital eccentricity (columns 28-35); (6) the argument of perigee in revolutions (columns 36-43); (7) the longitude (or right ascension) of the ascending node in revolutions (columns 44-51); (8) the orbital inclination in revolutions (columns 52-59); and (9) the epoch date (date of perigee passage) in the form **yyyymmdd**, where **yyyy** is the year, **mm** is the month, and **dd** is the day (columns 60-65). A decimal point must be present in columns 6, 14, 22, 28, 36, 44 and 52.

Select run mode 1 or 2:

1. Create an ephemeris.
2. Find times satellite is above the horizon.

Figure 2. Menu 2—Select run mode.

The **STATION.DAT** file must contain a single line of for each ground station or geographical location of interest. Each line must contain 5 entries, each enclosed in double quotations and separated by a blank space (Fig. 8). The entries are: (1) a station identification consisting of at most three alphanumeric characters; (2) the station latitude in the form **dd.mmss** (minus if south), where **dd** is degrees, **mm** is minutes, and **ss** is seconds; (3) the station longitude in the form **ddd.mmss** (minus if west); (4) the station altitude in feet (minus if below sea level); and (5) the name of the station or station location (alphanumeric).

If a data file input option is chosen, then SATSTA will open the two files described above and compute either an ephemeris or the times that each satellite is above the horizon (Option 1 or 2 of the second menu) for each ground station. Otherwise, keyboard input for one satellite and one ground station will be requested from the fourth and fifth menues, respectively.

¹The Herg—a unit of time named after the astronomer Paul Herget—is defined as the period, divided by 2π , of a fictitious satellite at zero altitude over a smooth spherical earth with no atmosphere.

```

Simulation-Time Data:

Starting date (yyyy.mmdd) ? 1983.0201
Starting time (hh.mmss) ? 00.00
Ending date (yyyy.mmdd) ? 1983.0202
Ending time (hh.mmss) ? 00.00
Time increment (minutes) ? 01

```

Press any key to continue.

Figure 3. Menu 3—Select simulation times.

Five pieces of information regarding the simulation are requested from the third menu (Fig. 3): (1) the simulation starting date in the form *yyyy.mmdd*, (2) the simulation starting time in the form *hh.mmss*; (3) the simulation ending date in the form *yyyy.mmdd*; (4) the simulation ending time in the form *hh.mmss*; and (5) the simulation time-step increment in minutes. The screen will display sample data for each entry in the appropriate format. If any sample entry is to be changed, it should be *completely* overwritten in order for it to be read in properly. When each entry is correct, enter it into the computer by pressing the \leftarrow key.

If the keyboard entry option was chosen from the first menu, then two more menus will appear, otherwise the simulation will commence with input being obtained from the two data files.

```

Satellite Data:

Epoch date (yyyy.mmdd) ? 1983.0201
Epoch time (hh.mmss) ? 00.00

Input mean elements of epoch:
Satellite ID Number ? 11111
Eccentricity ? 0.0005545
Long. ascending node (deg) ? 272.43497
Inclination (deg) ? 65.06057
Arg. of perigee (deg) ? 295.41470
Mean anomaly (deg) ? 258.10682
Mean motion (rev/day) ? 15.44194
Decay (rev/day^2) ? 0

```

Press any key to continue.

Figure 4. Menu 4—Input satellite data.

Menu 4 (Fig. 4) requires ten separate entries: (1) the epoch date (date of perigee passage) in the form *yyyy.mmdd*, where *yyyy* is the year, *mm* is the month, and *dd* is the day; (2) the epoch time (time of perigee passage) in the form *hh.mmss*, where *hh* is the hour, *mm* is the minutes, and *ss* is the seconds; (3) a satellite identification consisting of at most five alphanumeric characters; (4) the orbital eccentricity; (5) the longitude (or right ascension) of the ascending node in degrees; (6) the orbital inclination in degrees; (7) the argument of perigee in degrees; (8) the mean anomaly in degrees; (9) the mean motion in

revolutions per day; and (10) the orbital decay in revolutions per day-squared. The screen will display sample data for each entry in the appropriate format. Again, if any sample entry is to be changed, it should be *completely* overwritten in order for it to be read in properly. When each entry is correct, enter it into the computer by pressing the \leftarrow key.

Station Coordinates:

Station Identification Number	? 001
Station Location or Name	? Daisy, Tenn.
Station latitude (dd.mmss, South minus)	? 35.12
Station longitude (ddd.mmss, West minus)	? -85.12
Station altitude (feet)	? 500.0

Press any key to continue.

Figure 5. Menu 5—Input ground station data.

Menu 5 (Fig. 5) requires five separate entries: (1) a station identification consisting of at most three alphanumeric characters; (2) a station location or name; (3) the station latitude in the form dd.mmss (minus if south); (4) the station longitude in the form ddd.mmss (minus if west); and (5) the station altitude in feet (minus if below sea level).

C. Output. Output is written to the screen and is appended to the data file OUTPUT.DAT if it exists, otherwise file OUTPUT.DAT is created. A sample from the OUTPUT.DAT file is shown in Fig. 9—the structure of this file is discussed later.

Sample output from the “create an ephemeris” option screen is shown in Fig. 10. The apogee/perigee output is approximately valid only for the first orbit. In particular, the longitude is not corrected for the earth’s rotation during the current orbit. Also, the ascending node and argument of perigee are not corrected for their rates of change during the current orbit. It should be further noted that for nearly circular orbits, the height of the satellite will probably get lower than the perigee value and higher than the apogee value at locations other than the perigee and apogee positions—this is an effect of the earth’s oblateness.

The main body of the ephemeris output consists of date, time, satellite latitude, longitude and height, followed by the elevation angle, azimuth, and range (in kilometers) of the satellite from the station. The satellite “look angle” is the angle between the satellite “ground-zero” point (usually called the satellite “sub-point”) and the station. The last column of output is the satellite heading.

A sample of the output from the “times satellite is above the horizon” option screen for a time-step (Δt) of 1 minute is shown in Fig. 11. First the time of culmination for the current orbit is computed and saved. The culmination time is then incremented and decremented by Δt until the satellite is below the horizon. The time of rise and set is

"11111" "1983.0201" "0.00" "0.0005545" "272.43497" "65.06057" "295.41470" "258.10682" "16.44194" "0"
 "22222" "1983.0201" "0.00" "0.0023064" "55.552464" "64.94868" "202.277268" "126.6708986" "16.06302" "4.281119d-3"

Figure 6. Sample `elements.dat` file.

```
11111 7169634.9060250.00000.0005645.8205964.75667638.1807238830201
22222.3518636.9424655.23456.0023064.5618813.1543124.1804130830201
```

Figure 7. Sample `elements.nss` file.

```
"001" "35.12" "-85.12" "500.0" "Daisy, Tenn."
"002" "19.44" "-155.05" "300.0" "Hilo, Hawaii"
```

Figure 8. Sample `station.dat` file.

```
"001" "11111" "1983" "2" "1" "1" "13" "19.4" "54.37" "263.10" "450.1" "0.00" "340.48" "2436" "69.23" "136.43"
"001" "11111" "1983" "2" "1" "1" "18" "42.1" "38.12" "280.62" "443.8" "32.24" "55.33" "773" "52.35" "150.23"
"001" "11111" "1983" "2" "1" "1" "24" "3.8" "19.99" "290.85" "437.5" "-0.00" "132.74" "2400" "69.26" "156.58"
"001" "11111" "1983" "2" "1" "2" "49" "40.9" "45.38" "250.95" "446.7" "0.00" "306.66" "2428" "69.23" "145.41"
"001" "11111" "1983" "2" "1" "2" "54" "12.9" "30.63" "261.84" "441.0" "11.96" "251.45" "1429" "66.15" "153.58"
"001" "11111" "1983" "2" "1" "2" "58" "269.44" "15.05" "269.44" "436.2" "-0.00" "195.12" "2394" "69.26" "167.45"
"001" "22222" "1983" "2" "1" "1" "42" "46.3" "19.63" "268.42" "273.2" "0.00" "201.99" "1883" "73.38" "23.59"
"001" "22222" "1983" "2" "1" "1" "47" "1.1" "34.59" "276.49" "277.0" "58.93" "111.16" "321" "29.58" "28.27"
"001" "22222" "1983" "2" "1" "1" "51" "14.7" "48.49" "288.39" "280.2" "-0.00" "32.74" "1910" "73.46" "37.64"
"001" "22222" "1983" "2" "1" "3" "16" "59.0" "35.68" "254.51" "277.3" "0.00" "277.85" "1902" "73.40" "28.78"
"001" "22222" "1983" "2" "1" "3" "19" "25.5" "43.84" "260.89" "279.3" "3.08" "313.11" "1595" "73.18" "33.65"
"001" "22222" "1983" "2" "1" "3" "21" "50.0" "51.32" "269.26" "280.7" "-0.00" "347.67" "1910" "73.47" "40.74"
"001" "22222" "1983" "2" "1" "9" "36" "38.1" "48.57" "287.14" "264.2" "0.00" "30.30" "1853" "73.93" "142.26"
"001" "22222" "1983" "2" "1" "9" "37" "38.2" "45.41" "290.53" "261.9" "0.43" "44.49" "1798" "73.96" "145.11"
"001" "22222" "1983" "2" "1" "9" "38" "34.4" "42.35" "293.35" "259.8" "-0.00" "67.82" "1839" "73.99" "147.39"
"002" "11111" "1983" "2" "1" "5" "57" "53.0" "40.06" "208.38" "444.6" "0.00" "7.44" "2417" "69.34" "149.12"
"002" "11111" "1983" "2" "1" "6" "2" "14.5" "25.51" "217.41" "439.2" "9.78" "60.03" "1556" "67.27" "165.24"
"002" "11111" "1983" "2" "1" "6" "6" "34.7" "10.45" "224.16" "435.2" "-0.00" "112.97" "2395" "69.37" "158.01"
"002" "11111" "1983" "2" "1" "7" "32" "51.5" "34.44" "188.76" "442.4" "0.00" "319.38" "2413" "69.36" "152.04"
"002" "11111" "1983" "2" "1" "7" "38" "0.4" "16.85" "197.87" "436.6" "24.13" "250.20" "936" "58.62" "157.16"
"002" "11111" "1983" "2" "1" "7" "43" "8.8" "-1.18" "205.16" "433.8" "-0.00" "179.39" "2385" "69.39" "158.52"
"002" "22222" "1983" "2" "1" "4" "39" "23.2" "9.94" "218.83" "271.1" "0.00" "123.17" "1877" "73.53" "22.18"
"002" "22222" "1983" "2" "1" "4" "40" "29.5" "13.95" "220.61" "271.9" "0.58" "107.75" "1818" "73.52" "22.64"
"002" "22222" "1983" "2" "1" "4" "41" "36.5" "17.98" "222.27" "272.8" "-0.00" "92.22" "1885" "73.52" "23.27"
"002" "22222" "1983" "2" "1" "6" "8" "3.1" "6.39" "194.69" "270.5" "0.00" "218.92" "1873" "73.54" "21.90"
"002" "22222" "1983" "2" "1" "6" "12" "9.5" "21.23" "201.07" "273.6" "28.62" "296.75" "536" "57.38" "23.93"
"002" "22222" "1983" "2" "1" "6" "16" "16.0" "35.66" "209.09" "277.3" "-0.00" "11.90" "1896" "73.54" "28.77"
```

Figure 9. Sample `output.dat` file (some output expunged). Epoch of date.

Satellite: 11111 Station: 001 Daisy, Tenn.

Apogee:

Height = 442.8 km.
 Latitude = -55.1540
 Longitude = 100.2099

Perigee:

Height = 450.4 km.
 Latitude = 55.1539
 Longitude = 280.2099

Tue 1 Feb 1983

year	mo	da	hr	mn	sec	lat	long	height		look			
								(km)	elev	azim	range	angle	head
1983	2	1	0	0	0.0	-12.26	327.56	434.0	-31.60	123.30	7437	52.90	157.82
1983	2	1	0	10	0.0	-45.84	347.68	441.5	-50.59	136.39	10382	36.56	145.02
1983	2	1	0	20	0.0	-65.19	48.39	446.4	-69.07	151.85	12354	19.68	88.46
1983	2	1	0	30	0.0	-44.71	106.99	439.1	-83.25	221.39	13089	6.45	34.07
1983	2	1	0	40	0.0	-10.95	126.55	430.7	-70.51	302.57	12481	18.11	22.04
1983	2	1	0	50	0.0	24.08	141.07	435.9	-51.76	316.67	10574	35.26	24.38
1983	2	1	1	0	0.0	55.55	168.97	449.2	-32.06	324.49	7561	52.31	46.47

Figure 10. Output from ephemeris option. Epoch of date.

Satellite: 11111 Station: 001 Daisy, Tenn.

year	mo	da	hr	mn	sec	lat	long	height		look			
								(km)	elev	azim	range	angle	head
1983	2	1	1	13	19.4	54.37	263.10	450.1	0.00	340.48	2436	69.23	135.43
1983	2	1	1	13	42.1	53.35	264.76	449.7	1.39	341.68	2285	69.19	136.92
1983	2	1	1	14	42.1	50.53	268.75	448.7	5.55	345.74	1892	68.57	140.46
1983	2	1	1	15	42.1	47.58	272.27	447.6	10.78	351.87	1514	66.79	143.50
1983	2	1	1	16	42.1	44.51	275.37	446.4	17.70	2.15	1167	63.08	146.09
1983	2	1	1	17	42.1	41.35	278.14	445.1	26.48	21.39	892	56.95	148.32
1983	2	1	1	18	42.1	38.12	280.62	443.8	32.24	55.33	773	52.35	150.23
1983	2	1	1	19	42.1	34.83	282.88	442.6	27.03	90.21	875	56.40	151.86
1983	2	1	1	20	42.1	31.49	284.94	441.3	18.10	110.45	1141	62.69	153.26
1983	2	1	1	21	42.1	28.11	286.84	440.1	10.97	121.17	1484	66.60	154.46
1983	2	1	1	22	42.1	24.69	288.61	438.9	5.61	127.48	1862	68.51	155.46
1983	2	1	1	23	42.1	21.25	290.27	437.9	1.36	131.58	2255	69.20	156.31
1983	2	1	1	24	3.8	19.99	290.85	437.5	-0.00	132.74	2400	69.25	156.58

Figure 11. Output from "times above horizon" option. $\Delta t = 1$ minute. Epoch of date.

iterated until the time at which the satellite is on the horizon. All of these times and positions are then output to the screen. Note that the information printed out in this option, with the exception of the apogee/perigee positions, is identical to that printed out in the "ephemeris" option. By making Δt large, say 10 minutes, in the "times above horizon" option, then only the times of satellite rise, culmination, and set are computed and output (see Fig. 12).

Each line in the OUTPUT.DAT file consists of the station identifier and the satellite identifier followed by the same data, and in the same order, as that printed to the screen. This can be seen by comparing the first six lines of Fig. 12 with the same lines in Fig. 9.

Satellite: 11111 Station: 001 Daisy, Tenn.

year	mo	da	hr	mn	sec	lat	long	height	look				
								(km)	elev	azim	range	angle	head
1983	2	1	1	13	19.4	54.37	263.10	450.1	0.00	340.48	2436	69.23	135.43
1983	2	1	1	18	42.1	38.12	280.62	443.8	32.24	55.33	773	52.35	150.23
1983	2	1	1	24	3.8	19.99	290.85	437.5	-0.00	132.74	2400	69.25	156.58
1983	2	1	2	49	40.9	45.38	250.95	446.7	0.00	306.66	2428	69.23	145.41
1983	2	1	2	54	12.9	30.63	261.84	441.0	11.96	261.45	1429	66.15	153.58
1983	2	1	2	58	44.7	15.05	269.44	436.2	-0.00	195.12	2394	69.26	157.45
1983	2	1	14	47	13.6	16.76	285.33	433.6	0.00	150.29	2387	69.32	22.83
1983	2	1	14	50	58.0	29.64	291.62	437.9	6.44	106.44	1792	68.35	26.06
1983	2	1	14	54	43.3	42.02	300.04	443.2	-0.00	63.10	2419	69.28	32.12
1983	2	1	16	20	48.3	17.88	262.22	433.9	0.00	216.51	2389	69.32	23.02
1983	2	1	16	26	16.7	36.50	272.26	440.8	55.22	303.76	528	32.33	28.94
1983	2	1	16	31	46.6	53.37	289.30	448.2	-0.00	24.72	2431	69.26	43.11
1983	2	1	17	59	48.3	37.38	249.28	441.2	0.00	283.77	2414	69.29	29.38
1983	2	1	18	2	51.4	47.05	257.60	445.5	3.50	317.85	2066	69.01	36.02
1983	2	1	18	5	55.6	55.73	269.75	449.2	-0.00	351.87	2433	69.26	46.77

Figure 12. Output from "times above horizon" option. $\Delta t = 10$ minutes. Epoch of date.

Fig. 13 is a sample output obtained using the NAVSPASUR reference time. A comparison of Fig. 10 and Fig. 13 shows that the satellite latitude, height, and heading is the same but the longitude differs because of the difference of the earth's rotation between the two different reference times. The satellite-station relationships differ for the same reason. Figures 12 and 14 are completely dissimilar because of the differing satellite-station relationships.

Satellite: 11111 Station: 001 Daisy, Tenn.

Apogee:
Height = 442.8 km.
Latitude = -55.1540
Longitude = 230.8483

Perigee:
Height = 450.4 km.
Latitude = 55.1539
Longitude = 50.8483

Tue 1 Feb 1983

year	mo	da	hr	mn	sec	lat	long	height	look				
								(km)	elev	azim	range	angle	head
1983	2	1	0	0	0.0	-12.26	98.20	434.0	-77.96	351.78	12919	11.09	157.82
1983	2	1	0	10	0.0	-45.84	118.32	441.5	-79.40	232.53	12965	10.02	145.02
1983	2	1	0	20	0.0	-65.19	179.03	446.4	-60.98	210.44	11620	27.15	88.46
1983	2	1	0	30	0.0	-44.71	237.62	439.1	-41.32	205.69	9023	44.80	34.07
1983	2	1	0	40	0.0	-10.95	257.18	430.7	-20.38	203.40	5458	61.42	22.04
1983	2	1	0	50	0.0	24.08	271.70	435.9	12.73	194.92	1373	65.77	24.38
1983	2	1	1	0	0.0	55.55	299.61	449.2	-5.07	32.07	3062	68.64	46.47

Figure 13. Output from ephemeris option. NAVSPASUR reference time.

Satellite: 11111 Station: 001 Daisy, Tenn.

year	mo	da	hr	mn	sec	lat	long	height	look				
								(km)	elev	azim	range	angle	head
1983	2	1	0	47	32.1	15.54	267.79	433.2	0.00	199.85	2385	69.32	22.63
1983	2	1	0	52	59.8	34.24	277.36	439.8	58.70	112.56	509	29.01	27.87
1983	2	1	0	58	28.6	51.41	293.03	447.4	-0.00	33.16	2429	69.27	40.56
1983	2	1	2	24	57.1	29.86	251.15	438.0	0.00	262.00	2405	69.30	26.14
1983	2	1	2	29	10.7	43.72	260.91	443.9	8.69	312.74	1641	67.65	33.31
1983	2	1	2	33	25.7	55.99	276.83	449.3	-0.00	3.07	2433	69.26	47.22
1983	2	1	8	58	59.6	56.01	272.80	450.7	0.00	356.74	2437	69.23	132.74
1983	2	1	9	3	16.0	43.68	288.80	446.0	8.83	47.32	1637	67.55	146.72
1983	2	1	9	7	31.7	29.71	298.62	440.7	-0.00	98.40	2412	69.24	153.92
1983	2	1	10	33	57.2	51.41	256.65	449.0	0.00	326.74	2433	69.23	139.44
1983	2	1	10	39	26.8	34.21	272.34	442.3	58.17	247.39	514	29.48	152.14
1983	2	1	10	44	55.7	15.45	281.93	436.3	-0.00	160.33	2394	69.26	157.39
1983	2	1	12	13	31.5	31.47	250.42	441.3	0.00	266.88	2414	69.24	153.27
1983	2	1	12	14	27.8	28.30	252.20	440.2	0.29	256.89	2378	69.24	154.39
1983	2	1	12	15	23.2	25.15	253.85	439.1	-0.00	247.03	2407	69.25	155.34

Figure 14. Output from "times above horizon" option. $\Delta t = 10$ minutes. NAVSPASUR reference time.

III. SOURCES

As mentioned in the introduction, the high resolution and simplified satellite kinematics models were obtained from NAVSPASUR [Ref. 1]. The high resolution model is based upon a paper by Brouwer [Ref. 2]. The code for the high resolution model is given Appendix E, while the code for the simplified model is contained in the pos.update subroutine in Appendix D. Both of these routines require a solution of Kepler's equation. A non-iterative state-of-the-art solution of Kepler's equation, based upon a paper by Mikkola [Ref. 6] is contained in subroutine kepler listed in Appendix D.

The ground track determination is essentially that given by Escobal [Ref. 3, Ch. 10.3.2] but with a minor improvement described by Shudde [Ref. 7].

The method of computation of the times of satellite culmination, rise, and set are given in Appendix B, and the computation of the satellite heading is contained in Appendix C. Additional references are given there.

The IAU (1976) System of Astronomical Constants [Ref. 10, pg. S7] is used in SATSTA as well as the J2000.0 reference frame [Ref. 10, pg. L1]. Satellite positions computed by NAVSPASUR's SHOWALL [Ref. 1] use the WGS 72 constants and the J1900.0 reference frame. For this reason, the agreement between SATSTA and SHOWALL is not exact.

IV. DISCUSSION

Although the position updating in SATSTA is based upon the satellite kinematics model of Brouwer [Ref. 2], other models are currently in use. Most notable is the work of Hoots

and Roehrich [Ref. 5]. SATRAK [Ref. 4] is a FORTRAN program implementing their models and is available from the Aerospace Defense Command, Peterson AFB. Unfortunately, the source code is not available for modification.

The program presented here can be used as a basis for a more sophisticated model. The input scheme could be improved by using an input editor to catch improper types of entry and to allow corrections or changes to be made to previous entries. File management programs can be used to select satellite elements and station parameters from master files and insert them into "working" files for program input.

Various sensor types have not been modelled and no provision has been made for including them in the program. Also, satellite-satellite visibility has not been modelled. If such features are desirable, they could be included by modifying the existing program.

APPENDIX A. THE SIMPLIFIED KINEMATICS ALGORITHM.

The computation of the perturbed satellite orbit about the earth is performed using a method which is an approximation of that used by NAVSPASUR [Ref. 1]. According to NAVSPASUR, this approximation will yield satellite positons with a random error of about 10-15 nautical miles for a 20 day interval around epoch.

The mean elements at epoch, which must be input, are:

Eccentricity e_0

Right ascension of the ascending node Ω_0

Inclination i_0

Argument of perigee ω_0

Mean anomaly M_0

Mean motion M_1 , and

Decay M_2

Using the mean elements, the following auxillary quantities are computed:

$$a_0 = M_1^{-2/3}, \quad (\text{A-1})$$

$$a_0 = a_0 \left[1 + \frac{C(3\cos^2 i_0 - 1)}{a_0^2(1 - e_0^2)^{3/2}} \right]^{2/3}, \quad (\text{A-1a})$$

$$\dot{\omega} = C \frac{5\cos^2 i_0 - 1}{a_0^2(1 - e_0^2)^2}, \quad (\text{A-2})$$

$$\dot{\Omega} = -2C \frac{\cos i_0}{a_0^2(1 - e_0^2)^2}, \quad \text{and} \quad (\text{A-3})$$

$$\dot{a} = -\frac{4a_0}{3M_1} M_2, \quad (\text{A-4})$$

where $C = 3/4J_2$ is related to the oblateness of the earth. NAVSPASUR states the following:

For optimal accuracy, a_0 from Equation A-1 is used in Equation A-1a to obtain a final value for a_0 . This latter a_0 is then used throughout. Where program size is a major consideration and additional errors of some ten miles in satellite height can be tolerated the user may choose to leave out Equation A-1a and/or Equation A-4.

Using the mean values of the elements at epoch T_0 and the results of the above calculations, the mean values of the elements for any time t are:

$$M = M_0 + M_1(t - T_0) + M_2(t - T_0)^2,$$

$$e = e_0,$$

$$\omega = \omega_0 + \dot{\omega}(M - M_0),$$

$$\Omega = \Omega_0 + \dot{\Omega}(M - M_0) - \omega_e(t - T_0),$$

$$i = i_0, \quad \text{and}$$

$$a = a_0 + \dot{a}(t - T_0),$$

where ω_e is the earth's sidereal rotation rate.

APPENDIX B. THE TIME OF CULMINATION, RISE AND SET ALGORITHM.

In this program the earth is considered to be non-rotating, i.e., the coördinates of a station are given by latitude and longitude with the longitude unaffected by the earth's rotation. Rotation is accomplished by subtracting the accrued rotation from the right ascension of the ascending node of the satellite. Although this departs from realism, the benefit is that the rectangular coördinates of the station need to be calculated only once.

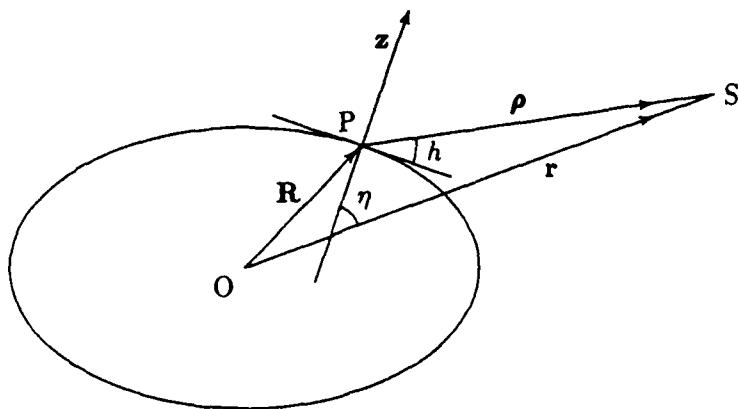


Fig. B-1. Satellite/Station Geometry.

Referring to Fig. B-1, culmination of the satellite, S, with respect to a ground station, P, occurs when the elevation angle, h , of the satellite is a maximum, which is equivalent to the minimization of the zenith angle, $90^\circ - h$, which is also equivalent to the minimization of the angle η between \mathbf{z} , the unit vector normal to the station location, and \mathbf{r} , the vector from the earth's center, O, to the satellite. The equation for η is

$$\cos \eta = \frac{\mathbf{z} \cdot \mathbf{r}}{r}, \quad (\text{B-1})$$

where \mathbf{z} is given by

$$\mathbf{z} = \begin{bmatrix} \cos \phi \cos \lambda \\ \cos \phi \sin \lambda \\ \sin \phi \end{bmatrix}, \quad (\text{B-2})$$

and ϕ and λ are the station geodetic latitude and longitude, respectively. The satellite position vector \mathbf{r} is obtained from [Ref. 3, Equ. (3.57)]

$$\mathbf{r} = x_\omega \mathbf{P} + y_\omega \mathbf{Q}, \quad (\text{B-3})$$

where x_ω and y_ω measure the satellite position in the orbital plane.¹ \mathbf{P} and \mathbf{Q} are the first two columns of the Euler matrix [Ref. 3, Eqns. (3.43) and (3.44)]—they transform the in-plane x_ω and y_ω coordinates to the equatorial rectangular system and are given by

$$\mathbf{P} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos \omega \cos \Gamma - \sin \omega \sin \Gamma \cos i \\ \cos \omega \sin \Gamma + \sin \omega \cos \Gamma \cos i \\ \sin \omega \sin i \end{bmatrix} \quad (\text{B-4a})$$

and

$$\mathbf{Q} = \begin{bmatrix} Q_x \\ Q_y \\ Q_z \end{bmatrix} = \begin{bmatrix} -\sin \omega \cos \Gamma - \cos \omega \sin \Gamma \cos i \\ -\sin \omega \sin \Gamma + \cos \omega \cos \Gamma \cos i \\ \cos \omega \sin i \end{bmatrix}, \quad (\text{B-4b})$$

where ω is the argument of perigee and i is the orbital inclination; $\Gamma = \Omega - \omega_e(t - t_0)$, where Ω is the longitude of the ascending node at the time, t_0 , of the last element update; ω_e is the earth's sidereal rotation rate and t is the current time.

The components of \mathbf{r} vary with time. We will transform the equations in such a way that will replace time by the eccentric anomaly E as the independent variable. The inclination i is time independent and the elements Ω and ω will be considered to be independent of time over the period of a single orbit even though they are time dependent through perturbations. The components of \mathbf{r} which will be considered to be time dependent are then x_ω , y_ω and Γ .

Write Equation (B-1) as

$$\mathbf{z} \cdot \mathbf{r} - r \cos \eta = 0.$$

Take the derivative with respect to E to obtain,

$$\mathbf{z} \cdot \frac{d\mathbf{r}}{dE} - \frac{dr}{dE} \cos \eta + r \sin \eta \frac{d\eta}{dE} = 0.$$

To find the value of E which minimizes η , set $d\eta/dE = 0$. Thus,

$$\mathbf{z} \cdot \frac{d\mathbf{r}}{dE} - \frac{dr}{dE} \cos \eta = 0,$$

or

$$\mathbf{z} \cdot \frac{d\mathbf{r}}{dE} - \frac{1}{r} \frac{dr}{dE} \mathbf{z} \cdot \mathbf{r} = 0.$$

Since this equation cannot be solved explicitly for E , set

$$F(E) = \mathbf{z} \cdot \frac{d\mathbf{r}}{dE} - \frac{1}{r} \frac{dr}{dE} \mathbf{z} \cdot \mathbf{r} \quad (\text{B-5})$$

¹The origin of the system is at the focus of the orbit. x_ω is positive in the direction of the perifocus. y_ω lies in the orbital plane and is orthogonal to x_ω .

and find E such that $F(E) = 0$ by the Newton-Raphson method. The Newton-Raphson procedure also requires $F'(E)$, the derivative of $F(E)$ with respect to E :

$$F'(E) = \mathbf{z} \cdot \frac{d^2\mathbf{r}}{dE^2} + \frac{1}{r^2} \left(\frac{dr}{dE} \right)^2 - \frac{1}{r} \frac{d^2r}{dE^2} \mathbf{z} \cdot \mathbf{r} - \frac{1}{r} \frac{dr}{dE} \mathbf{z} \cdot \frac{dr}{dE}. \quad (\text{B-6})$$

The first and second derivatives of Equation (B-3) are

$$\frac{d\mathbf{r}}{dE} = \frac{dx_\omega}{dE} \mathbf{P} + x_\omega \frac{d\mathbf{P}}{dE} + \frac{dy_\omega}{dE} \mathbf{Q} + y_\omega \frac{d\mathbf{Q}}{dE} \quad (\text{B-7a})$$

and

$$\frac{d^2\mathbf{r}}{dE^2} = \frac{d^2x_\omega}{dE^2} \mathbf{P} + 2 \frac{dx_\omega}{dE} \frac{d\mathbf{P}}{dE} + x_\omega \frac{d^2\mathbf{P}}{dE^2} + \frac{d^2y_\omega}{dE^2} \mathbf{Q} + 2 \frac{dy_\omega}{dE} \frac{d\mathbf{Q}}{dE} + y_\omega \frac{d^2\mathbf{Q}}{dE^2}, \quad (\text{B-7b})$$

where [Ref. 3, Equ. (3.69)]

$$\begin{aligned} x_\omega &= a(\cos E - e) \quad \text{and} \\ y_\omega &= a\sqrt{1 - e^2} \sin E, \end{aligned} \quad (\text{B-8a})$$

with first and second derivatives

$$\begin{aligned} \frac{dx_\omega}{dE} &= -a \sin E, \\ \frac{dy_\omega}{dE} &= a\sqrt{1 - e^2} \cos E, \end{aligned} \quad (\text{B-8b})$$

and

$$\begin{aligned} \frac{d^2y_\omega}{dE^2} &= -a\sqrt{1 - e^2} \sin E = -y_\omega, \\ \frac{d^2x_\omega}{dE^2} &= -a \cos E. \end{aligned} \quad (\text{B-8c})$$

The first and second derivatives of Equations (B-4) are

$$\begin{aligned} \frac{dP_x}{dE} &= -P_y \frac{d\Gamma}{dE}, \\ \frac{dP_y}{dE} &= P_x \frac{d\Gamma}{dE}, \\ \frac{dP_z}{dE} &= 0, \\ \frac{dQ_x}{dE} &= -Q_y \frac{d\Gamma}{dE}, \\ \frac{dQ_y}{dE} &= Q_x \frac{d\Gamma}{dE}, \\ \frac{dQ_z}{dE} &= 0, \end{aligned} \quad (\text{B-9a})$$

and

$$\begin{aligned}
 \frac{d^2 P_x}{dE^2} &= -P_x \left(\frac{d\Gamma}{dE} \right)^2 - P_y \frac{d^2 \Gamma}{dE^2}, \\
 \frac{d^2 P_y}{dE^2} &= -P_y \left(\frac{d\Gamma}{dE} \right)^2 + P_x \frac{d^2 \Gamma}{dE^2}, \\
 \frac{d^2 P_z}{dE^2} &= 0, \\
 \frac{d^2 Q_x}{dE^2} &= -Q_x \left(\frac{d\Gamma}{dE} \right)^2 - Q_y \frac{d^2 \Gamma}{dE^2}, \\
 \frac{d^2 Q_y}{dE^2} &= -Q_y \left(\frac{d\Gamma}{dE} \right)^2 + Q_x \frac{d^2 \Gamma}{dE^2}, \quad \text{and} \\
 \frac{d^2 Q_z}{dE^2} &= 0.
 \end{aligned} \tag{B-9b}$$

Also required is [Ref. 3, Equ. (3.67)]

$$r = a(1 - e \cos E) \tag{B-10a}$$

and it's derivatives

$$\begin{aligned}
 \frac{dr}{dE} &= ae \sin E, \quad \text{and} \\
 \frac{d^2 r}{dE^2} &= ae \cos E.
 \end{aligned} \tag{B-10b}$$

Finally, using Kepler's equation [Ref. 3, Equ. (3.79)],

$$t = \frac{E - e \sin E}{n} + T, \tag{B-11}$$

where n is the rate of orbital motion and T is the time of last perifocal passage, the equation for Γ can be written as

$$\Gamma = \Omega - \omega_e \left(\frac{E - e \sin E}{n} + T - t_0 \right). \tag{B-12a}$$

Then,

$$\frac{d\Gamma}{dE} = -\omega_e \left(\frac{1 - e \cos E}{n} \right), \tag{B-12b}$$

and

$$\frac{d^2 \Gamma}{dE^2} = \frac{-\omega_e(e \sin E)}{n}. \tag{B-12c}$$

To solve Equation (B-5), an initial estimate of E is made and the right-hand side of Equation (B-13) is evaluated. The result, the left-hand side of Equation (B-13), is an improved estimate of E ,

$$E = E - \frac{F(E)}{F'(E)}. \tag{B-13}$$

This iteration is continued until $|F(E)/F'(E)|$ is less than some prescribed amount. The value of E obtained will either maximize or minimize η . The desired value of η will be the one for which $\cos \eta > 0$.

Once the eccentric anomaly for culmination, E_c , is found, the time of culmination is determined directly from Equation (B-11). Perturbing E_c by plus or minus about 10° , it is possible to obtain good starting values for the determination of the satellite set and rise times, respectively.

The procedure for finding the rise and set times is very similar to finding the time of culmination. The rise/set equations are developed by Escobal [Ref. 3, Sec. 5.4], but the algorithm presented here is somewhat simplified since, unlike Escobal, the time of culmination is available. In fact, many of the same equations can be used. Again, the eccentric anomaly will be used as the independent variable. Referring to Fig. B-1, satellite rise or set occurs when the elevation angle h is equal to zero, where h can be found from the equation

$$\sin h = \frac{\rho \cdot \mathbf{z}}{\rho}, \quad (\text{B-14})$$

where

$$\rho = \mathbf{r} - \mathbf{R}. \quad (\text{B-15})$$

In Equation (B-15), \mathbf{R} is the station coordinate vector and ρ is the slant range vector from the station to the satellite. For the spheroid earth [Ref. 3, Equ. (5.54)],

$$\mathbf{R} = \begin{bmatrix} G_1 \cos \phi \cos \lambda \\ G_1 \cos \phi \sin \lambda \\ G_2 \sin \phi \end{bmatrix}, \quad (\text{B-16})$$

where ϕ and λ are the station geodetic latitude and longitude, respectively. G_1 and G_2 are factors which account for the earth's oblateness, and are given by

$$G_1 = \frac{a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi}} + H \quad \text{and} \\ G_2 = \frac{(1 - f^2)a_e}{\sqrt{1 - (2f - f^2) \sin^2 \phi}} + H, \quad (\text{B-17})$$

where

a_e = earth's equatorial radius,

f = earth's flattening factor, and

H = station elevation above the ellipsoid.

Write Equation (B-14) as

$$\rho \cdot \mathbf{z} = \rho \sin h.$$

When the satellite is on the horizon, $h = 0$ and $\sin h = 0$. Since the resulting equation, $\rho \cdot \mathbf{z} = 0$, cannot be solved explicitly for E , set

$$G(E) = \rho \cdot \mathbf{z}$$

and find E such that $G(E) = 0$ by the Newton-Raphson method. Using Equation (B-15), write $G(E)$ as

$$G(E) = \mathbf{z} \cdot (\mathbf{r} - \mathbf{R}). \quad (\text{B-18})$$

The Newton-Raphson procedure also requires $G'(E)$, the derivative of $G(E)$ with respect to E . Since the earth is being treated as non-rotating with the sidereal rotation subtracted from the satellite's ascending node, \mathbf{R} is independent of E , hence

$$G'(E) = \mathbf{z} \cdot \frac{d\mathbf{r}}{dE}. \quad (\text{B-19})$$

In Equations (B-18) and (B-19), \mathbf{z} , \mathbf{r} , $d\mathbf{r}/dE$, and the additional needed derivatives have all been given previously in this section.

To solve Equation (B-18), an initial estimate of E is made and the right-hand side of Equation (B-20) is evaluated. The result, the left-hand side of Equation (B-20), is an improved estimate of E ,

$$E = E - \frac{G(E)}{G'(E)}. \quad (\text{B-20})$$

The iteration is continued until $|G(E)/G'(E)|$ is less than some prescribed amount. The corresponding time of rise or set is then obtained from Equation (B-11).

APPENDIX C. THE SATELLITE HEADING COMPUTATION.

A satellite can be located by a position vector $\mathbf{x} = x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k}$ in a rectangular coordinate system with origin at the earth's center. In matrix form, and with x_1 , x_2 and x_3 expressed in spherical coordinates,

$$\mathbf{x} = \begin{bmatrix} r \cos \phi \cos \lambda \\ r \cos \phi \sin \lambda \\ r \sin \phi \end{bmatrix}$$

where ϕ is the latitude, λ is the longitude at the point, and $r = |\mathbf{x}|$ is the distance of the point from the earth's center.

Let \mathbf{z} be the unit vector in the direction of \mathbf{r} . Then

$$\mathbf{z} = \frac{\mathbf{x}}{|\mathbf{x}|} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \cos \phi \cos \lambda \\ \cos \phi \sin \lambda \\ \sin \phi \end{bmatrix}.$$

It is convenient to define two other unit vectors in the plane tangent to \mathbf{z} . These are local north \mathbf{n} and local east \mathbf{e} , defined by

$$\mathbf{n} = \frac{\mathbf{z}_\phi}{|\mathbf{z}_\phi|} = \begin{bmatrix} -\sin \phi \cos \lambda \\ -\sin \phi \sin \lambda \\ \cos \phi \end{bmatrix} \quad \text{and} \quad \mathbf{e} = \frac{\mathbf{z}_\lambda}{|\mathbf{z}_\lambda|} = \begin{bmatrix} -\sin \lambda \\ \cos \lambda \\ 0 \end{bmatrix},$$

where

$$\mathbf{z}_\phi \equiv \frac{\partial \mathbf{z}}{\partial \phi} \quad \text{and} \quad \mathbf{z}_\lambda \equiv \frac{\partial \mathbf{z}}{\partial \lambda}.$$

The vectors \mathbf{z} , \mathbf{n} and \mathbf{e} provide the basis for a right-handed orthogonal coordinate system.

If a satellite's velocity vector $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is known at position \mathbf{x} then it is easily shown [Ref. 8] that

$$\mathbf{n} \cdot \mathbf{v} = \cos \alpha \quad \text{and}$$

$$\mathbf{e} \cdot \mathbf{v} = \sin \alpha,$$

where α is the course angle or heading. From these relations, α is found to be

$$\alpha = \text{qatn}(\mathbf{e} \cdot \mathbf{v}, \mathbf{n} \cdot \mathbf{v})$$

where $\text{qatn}(y, x)$ is $\arctan(y/x)$ corrected to the proper quadrant.

Since the position and velocity component's are in rectangular coordinates, it will be necessary to convert them to spherical coordinates in order to determine the components of \mathbf{n} and \mathbf{e} . However, the spherical coordinate step can be circumvented as follows: First,

$$\cos \alpha = \mathbf{n} \cdot \mathbf{v} = -v_1 \sin \phi \cos \lambda - v_2 \sin \phi \sin \lambda + v_3 \cos \phi.$$

Multiply both sides by $\cos \phi$ to give

$$\begin{aligned}
\cos \phi \cos \alpha &= -v_1 \sin \phi \cos \phi \cos \lambda - v_2 \sin \phi \cos \phi \sin \lambda + v_3 \cos^2 \phi \\
&= -\sin \phi(z_1 v_1 + z_2 v_2) + v_3 \cos^2 \phi + z_3 v_3 \sin \phi - z_3 v_3 \sin \phi \\
&= -\sin \phi(z_1 v_1 + z_2 v_2 + z_3 v_3) + v_3 \cos^2 \phi + z_3 v_3 \sin \phi \\
&= -z_3(\mathbf{z} \cdot \mathbf{v}) + v_3 \cos^2 \phi + v_3 \sin^2 \phi \\
&= v_3 - z_3(\mathbf{z} \cdot \mathbf{v}).
\end{aligned}$$

Similarly,

$$\sin \alpha = \mathbf{e} \cdot \mathbf{v} = -v_1 \sin \lambda + v_2 \cos \lambda,$$

so that

$$\begin{aligned}
\cos \phi \sin \alpha &= v_2 \cos \phi \cos \lambda - v_1 \cos \phi \sin \lambda \\
&= z_1 v_2 - z_2 v_1.
\end{aligned}$$

Then,

$$\alpha = \text{qatn}(\cos \phi \sin \alpha, \cos \phi \cos \alpha).$$

Since $\cos \phi > 0$ for all $\phi \in (-\pi/2, \pi/2)$, the $\cos \phi$ term effectively “cancels out” [Ref. 9] and so the heading is determined by

$$\begin{aligned}
\alpha &= \text{qatn}(\sin \alpha, \cos \alpha) \\
&= \text{qatn}(z_1 v_2 - z_2 v_1, v_3 - z_3(\mathbf{z} \cdot \mathbf{v})).
\end{aligned}$$

This equation is used by NAVSPASUR [Ref. 1] in SHOWALL. In SATSTA, the equivalent form,

$$\alpha = \text{qatn}\left(\frac{x_1 v_2 - x_2 v_1}{r}, v_3 - \frac{x_3(\mathbf{x} \cdot \mathbf{v})}{r^2}\right),$$

is used.

APPENDIX D. THE SATSTA PROGRAM LISTING.

```

'SATSTA
' Satellite Track. 03-03-88.           Rev. 09-26-89 @ 0830.

DECLARE FUNCTION acos# (x#)
DECLARE FUNCTION asin# (x#)
DECLARE FUNCTION atan2# (y#, x#)
DECLARE FUNCTION decimal# (v$)
DECLARE FUNCTION dmod# (x#, m#)
DECLARE FUNCTION dot# (a#(), b#())
DECLARE FUNCTION gmat# (tu#, tm#)
DECLARE SUB apo.peri.gee (elem#(), elemu#())
DECLARE SUB culmnate (zv#(), we#, t0#, ea#, coseta#, elemu#())
DECLARE SUB euler (peri#, node#, incl#, psn#, vel#())
DECLARE SUB ground.track (r#, sat.dc#, sat.lt#, sat.ht#)
DECLARE SUB hr.min.sec (time.of.day#, hr%, min%, sec%, n%)
DECLARE SUB jd.to.date (jd#(), m#, dk#, y#, h#, mo$, day$)
DECLARE SUB julian.day.number (m#, dk#, y#, ut#, jd#(), dj#, tj#)
DECLARE SUB kepler (m#, ec#, ea#)
DECLARE SUB output4 (k#, yr#, mok#, dtk#, time.of.day#, lt#, ln#, ht#, el#, _
                     az#, rng#, langle#, head#, sta.id$, sat.id$)
DECLARE SUB pos.update (1%, elem#(), time#, sat.x#(), sat.xd#(), elemu#(), _
                     r#, ea#)
DECLARE SUB rise.set (zv#(), sta.R#(), we#, t0#, ea#, elemu#())
DECLARE SUB sat.head (r#, sat.x#(), sat.xd#(), head#)
DECLARE SUB sat.sphere.coord (sat.x#(), r#, tj#, tm#, sat.dc#, sat.ln#, sat.ra#)
DECLARE SUB sta.p.coord (lt#, ln#, ht#, x#())
DECLARE SUB sta.sat (sta.x#(), sat.x#(), rng#, az#, el#, langle#)
DECLARE SUB time.update (k#, deltat#, tm#, jd#, jd#(), tj#, time.of.day#, _
                     m#, dk#, y#, h#, mo$, day$)
DECLARE SUB yrmaday (v$, yr#, mok#, dy#)

'COLOR 7, 1
'SCREEN 0, 1

DEFDBL A-Z
OPTION BASE 1
DIM jepoch(2), jdstart(2), jdend(2), jd(2), sat.x(3), sat.xd(3), sta.x(3, 4)
DIM elem(9), elemu(9), pv(3), qv(3), sta.z(3), sta.R(3), jd.charlie(2)

' constants
CONST pi = 3.141592653589793#, rad = pi / 180#, twopi = pi + pi
CONST j2 = .0010826318#
CONST nmi.ft = .3048# / 1852#   ' n.mi/foot
CONST ae = 6378.14#             ' earth's equatorial radius (km.)
CONST GE = 398600.5#            ' geo. grav. const. (km)^3/(sec)^2
k = 60# * SQR(GE / ae) / ae^(3/2)/min
Herg.min = k
CONST f1 = 1# / 298.257#        ' earth's flattening factor
CONST e.sqr = (2# - f1) * f1    ' earth's ellipticity squared
CONST we = 1.002737909350795# * twopi / 1440# 'earth's rotation (radians/minute)
CONST sixty = 60#

' Sample input (NORAD Project Spacetrack Rpt. #3, Dec. 1980):
amajor$ = "1.04050189": ecc$ = "0.0086731": aincl$ = "72.8435"
peri$ = "52.6988": anode$ = "115.9689": manomaly$ = "110.5714"
motion$ = "16.05824518": decay$ = "0"
epochdate$ = "1980.1001": epochtime$ = "23.41241138"
startdate$ = "1980.1001": starttime$ = "23.41241138"
enddate$ = "1980.1003": endtime$ = "0000"
deltat$ = "4"

' Sample input of arbitrary higher altitude, higher ecc orbit
ecc$ = "0.0200000": aincl$ = "64.94868"
peri$ = "202.277268": anode$ = "55.552464": manomaly$ = "126.670896"

```

```

motion$ = "12.00000": decay$ = "4.281098d-3"
epochdate$ = "1983.0201": epochtime$ = "00.00"
startdate$ = "1983.0201": starttime$ = "00.00"
enddate$ = "1983.0208": endtime$ = "00.00"
deltat$ = "15"
sta.lat$ = "35.12": sta.lon$ = "-85.12": sta.alt$ = "500.0"

' Sample input (NAVSPASUR: Sat. 22222 data from SHOWALL, private comm.
'   Mr. Fred Lipp, July 1988):
sat.id$ = "22222"
ecc$ = "0.0023064": ainc1$ = "64.94868"
peri$ = "202.277268": anode$ = "55.552464": manomaly$ = "126.670896"
motion$ = "16.06302": decay$ = "4.281119d-3"
epochdate$ = "1983.0201": epochtime$ = "00.00"
startdate$ = "1983.0201": starttime$ = "00.00"
enddate$ = "1983.0203": endtime$ = "00.00"
deltat$ = "01"
sta.id$ = "001": sta.name$ = "Daisy, Tenn."
sta.lat$ = "35.12": sta.lon$ = "-85.12": sta.alt$ = "500.0"

' Sample input (NAVSPASUR: Sat. 11111 data from SHOWALL, private comm.
'   Mr. Fred Lipp, July 1988):
sat.id$ = "11111"
ecc$ = "0.0005545": ainc1$ = "65.06057"
peri$ = "295.41470": anode$ = "272.43497": manomaly$ = "258.10682"
motion$ = "15.44194": decay$ = "0"
epochdate$ = "1983.0201": epochtime$ = "00.00"
startdate$ = "1983.0201": starttime$ = "00.00"
enddate$ = "1983.0202": endtime$ = "00.00"
deltat$ = "10"
sta.id$ = "001": sta.name$ = "Daisy, Tenn."
sta.lat$ = "35.12": sta.lon$ = "-85.12": sta.alt$ = "500.0"

start:
CLS
PRINT "Select data input mode 1, 2, 3 or 4:"
PRINT
PRINT SPC(5); "Keyboard input for satellite & ground station:"
PRINT SPC(10); "1. Epoch of date."
PRINT SPC(10); "2. NAVSPASUR Reference Date."
PRINT
PRINT SPC(5); "Data file input for satellite & ground station:"
PRINT SPC(10); "3. Epoch of date."
PRINT SPC(10); "4. NAVSPASUR One-Line Charlie."
data$ = ""
WHILE data$ < "1" OR data$ > "4"
    data$ = INPUT$(1)
WEND

CLS
PRINT "Select run mode 1 or 2:"
PRINT
PRINT SPC(5); "1. Create an ephemeris."
PRINT SPC(5); "2. Find times satellite is above the horizon."
case$ = ""
WHILE case$ <> "1" AND case$ <> "2"
    case$ = INPUT$(1)
WEND

GOSUB input.sim.times

SELECT CASE data$
CASE "1", "2"
    CLS
    PRINT "Satellite Data:"
    PRINT

```

```

PRINT "Epoch date (yyyy.mmdd)      ? "; epochdate$
LOCATE 3, 29: INPUT v$: IF v$ = "" THEN v$ = epochdate$
epochdate$ = v$: CALL yrmoday(v$, yr&, mo&, dy&)

PRINT "Epoch time (hh.mmss)      ? "; epochtime$
LOCATE 4, 29: INPUT v$: IF v$ = "" THEN v$ = epochtime$
epochtime$ = v$: epochtime = decimal$(v$)
CALL julian.day.number(mo&, dy&, yr&, epochtime, jepoch(), epoch, tepoch)

LOCATE 6, 1: PRINT "Input mean elements of epoch:"

PRINT "Satellite ID Number      ? "; sat.id$
LOCATE 7, 29: INPUT v$: IF v$ = "" THEN v$ = sat.id$
sat.id$ = v$

PRINT "Eccentricity      ? "; ecc$
LOCATE 8, 29: INPUT v$: IF v$ = "" THEN v$ = ecc$
ecc$ = v$: ecc = VAL(v$)

PRINT "Long. ascending node (deg)      ? "; anode$
LOCATE 9, 29: INPUT v$: IF v$ = "" THEN v$ = anode$
anode$ = v$: anode = VAL(v$) * rad

PRINT "Inclination (deg)      ? "; aincl$
LOCATE 10, 29: INPUT v$: IF v$ = "" THEN v$ = aincl$
aincl$ = v$: aincl = VAL(v$) * rad

PRINT "Arg. of perigee (deg)      ? "; peri$
LOCATE 11, 29: INPUT v$: IF v$ = "" THEN v$ = peri$
peri$ = v$: peri = VAL(v$) * rad

PRINT "Mean anomaly (deg)      ? "; manomaly$
LOCATE 12, 29: INPUT v$: IF v$ = "" THEN v$ = manomaly$
manomaly$ = v$: manomaly = VAL(v$) * rad

PRINT "Mean motion (rev/day)      ? "; motion$
LOCATE 13, 29: INPUT v$: IF v$ = "" THEN v$ = motion$
motion$ = v$: motion = VAL(v$) * twopi / 1440$' convert to radians/minute

PRINT "Decay (rev/day^2)      ? "; decay$
LOCATE 14, 29: INPUT v$: IF v$ = "" THEN v$ = decay$
' convert to radians/minute^2:
decay$ = v$: decay = VAL(v$) * twopi / (1440$) ^ 2

LOCATE 16, 1: PRINT "Press any key to continue."
FOR i% = 1 TO 10: c$ = INKEY$: NEXT
c$ = INPUT$(1)

CLS
LOCATE 1, 1: PRINT "Station Coordinates:"
PRINT

PRINT "Station Identification Number      ? "; sta.id$
LOCATE 3, 42: INPUT v$: IF v$ = "" THEN v$ = sta.id$
sta.id$ = v$

PRINT "Station Location or Name      ? "; sta.name$
LOCATE 4, 42: INPUT v$: IF v$ = "" THEN v$ = sta.name$
sta.name$ = v$: sta.lat = VAL(v$) * rad

PRINT "Station latitude (dd.mmss, South minus) ? "; sta.lat$
LOCATE 5, 42: INPUT v$: IF v$ = "" THEN v$ = sta.lat$
sta.lat$ = v$: sta.lat = VAL(v$) * rad

PRINT "Station longitude (ddd.mmss, West minus) ? "; sta.lon$

```

```

LOCATE 6, 42: INPUT v$: IF v$ = "" THEN v$ = sta.lon$
sta.lon$ = v$: sta.lon = VAL(v$) * rad

PRINT "Station altitude (feet) ? "; sta.alt$
LOCATE 7, 42: INPUT v$: IF v$ = "" THEN v$ = sta.alt$
sta.alt$ = v$: sta.alt = VAL(v$)

LOCATE 9, 1: PRINT "Press any key to continue."
FOR i% = 1 TO 10: c$ = INKEY$: NEXT
c$ = INPUT$(1)

OPEN "output.dat" FOR APPEND AS #3

GOSUB compute

CASE "3"
OPEN "station.dat" FOR INPUT AS #1
OPEN "output.dat" FOR APPEND AS #3

DO UNTIL EOF(1)
INPUT #1, sta.id$, sta.lat$, sta.lon$, sta.alt$, sta.name$
sta.lat = VAL(sta.lat$) * rad
sta.lon = VAL(sta.lon$) * rad
sta.alt = VAL(sta.alt$)

OPEN "elements.dat" FOR INPUT AS #2

DO UNTIL EOF(2)
INPUT #2, sat.id$, epochdate$, epochtime$, ecc$, anode$, aincl$, -
peri$, manomaly$, motion$, decay$

CALL yrmday(epochdate$, yr&, mo&, dy&)
epochtime = decimal$(epochtime$)
CALL julian.day.number(mo&, dy&, yr&, epochtime, jdepoch(), epoch, -
tjepoch)
ecc = VAL(ecc$)
anode = VAL(anode$) * rad
aincl = VAL(aincl$) * rad
peri = VAL(peri$) * rad
manomaly = VAL(manomaly$) * rad
motion = VAL(motion$) * twopi / 1440#' convert to radians/minute
decay = VAL(decay$) * twopi / (1440#) ^ 2 ' convert to rad/min^2

jd(1) = jdstart(1)
jd(2) = jdstart(2)
jd = jd(1) + jd(2)
time.of.day = starttime

GOSUB compute

LOOP
CLOSE 2
LOOP

CASE "4"
OPEN "station.dat" FOR INPUT AS #1
OPEN "output.dat" FOR APPEND AS #3

DO UNTIL EOF(1)
INPUT #1, sta.id$, sta.lat$, sta.lon$, sta.alt$, sta.name$
sta.lat = VAL(sta.lat$) * rad
sta.lon = VAL(sta.lon$) * rad
sta.alt = VAL(sta.alt$)

OPEN "elements.nss" FOR INPUT AS #2

```

```

DO UNTIL EOF(2)
  INPUT #2, c$

  sat.id$ = MID$(c$, 1, 5)
  manomaly$ = MID$(c$, 6, 9)
  motion$ = MID$(c$, 14, 9)
  decay$ = MID$(c$, 22, 7)
  ecc$ = MID$(c$, 28, 9)
  peri$ = MID$(c$, 36, 9)
  anode$ = MID$(c$, 44, 9)
  aincl$ = MID$(c$, 52, 9)
  epochdate$ = "19" + MID$(c$, 60, 2) + "." + MID$(c$, 62, 4)
  epochtime$ = "00.00"

  CALL yrmoday(epochdate$, yr$, mo$, dy$)
  epochtime = decimal$(epochtime$)
  CALL julian.day.number(mo$, dy$, yr$, epochtime, jdepoch(), epoch, -
    tjepoch)
  ecc = VAL(ecc$)
  anode = VAL(anode$) * twopi
  aincl = VAL(aincl$) * twopi
  peri = VAL(peri$) * twopi
  manomaly = VAL(manomaly$) * twopi
  motion = VAL(motion$) * Herg.min      ' convert to radians/minute
  decay = VAL(decay$)
  IF decay > .5# THEN decay = decay - 1#
  decay = decay * .00001 * Herg.min^2   ' convert to rad/min^2

  jd(1) = jdstart(1)
  jd(2) = jdstart(2)
  jd = jd(1) + jd(2)
  time.of.day = starttime

  GOSUB compute

  LOOP
  CLOSE 2
  LOOP
END SELECT
CLOSE

END

' * * * * *

compute:
' initialize times.
tj = tjstart          ' Julian centuries from J2000.0
begin.time = (startdate - epoch) * 1440# ' minutes
end.time = (enddate - epoch) * 1440# ' minutes

elem(1) = amajor      ' semi-major axis
elem(2) = ecc          ' eccentricity
elem(3) = tzero        ' time of perifocal passage
elem(4) = aincl        ' inclination (radians)
elem(5) = anode        ' longitude of ascending node (radians)
elem(6) = peri         ' argument of perifocus (radians)
elem(7) = motion        ' mean motion (revolutions/day)
elem(8) = manomaly      ' mean anomaly (radians)
elem(9) = decay         ' decay constant (radians/minute^2)

SELECT CASE data$
CASE "1", "3"
  ' Since the earth is treated as non-rotating, elem(5) must be corrected for
  ' the accrued rotation from epoch to J2000.0
  elem(5) = elem(5) - 15# * gmst(tjepoch, 0#) * rad

```

```

CASE "2", "4"
    ' NAVSPASUR one-line Charlie correction
    CALL julian.day.number(1, 1, 1985, 0, jd.charlie(), dummy, tj.charlie)
    wetc = (tj.epoch - tj.charlie) * 36525# - (1# - gmst(tj.charlie, 0#) / 24#) -
        * .9972695664#
    wetc = dmod(wetc * 1440# * we, twopi)
    elem(5) = elem(5) + wetc
    elem(5) = elem(5) - 15# * gmst(tj.epoch, 0#) * rad
END SELECT

CALL sta.p.coord(sta.lat, sta.lon, sta.alt, sta.x())
FOR i% = 1 TO 3
    sta.z(i%) = sta.x(i%, 2)
    sta.R(i%) = sta.x(i%, 1)
NEXT

CALL pos.update(1, elem(), begin.time, sat.x(), sat.xd(), elemu(), r, ea)
CALL pos.update(2, elem(), begin.time, sat.x(), sat.xd(), elemu(), r, ea)

CLS
PRINT "Satellite: "; sat.id$; "     Station: "; sta.id$; "   "; sta.name$
PRINT

SELECT CASE case$
'-----
CASE "1"

tm = starttime                         'UT in hours
CALL apo.peri.gee(elem(), elemu())

CALL jd.to.date(jd(), mk, dk, yk, h#, mo$, day$)
PRINT
PRINT day$; dk; mo$; yk

CALL output4(0, yr#, mo#, dk#, time.of.day, sat.lt, sat.ln, sat.ht, el, az, -
            rng, langle, head, sta.id$, sat.id$)

' time-loop
FOR time = begin.time TO end.time STEP delta.t
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)

    'compute spherical coordinates
    CALL sat.sphere.coord(sat.x(), r, tj, tm, sat.dc, sat.ln, sat.ra)

    'satellite heading
    CALL sat.head(r, sat.x(), sat.xd(), head)

    'earth ground track
    CALL ground.track(r, sat.dc, sat.lt, sat.ht)

    'compute station-satellite relations
    CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)

    rng = rng * ae

    CALL output4(1, yk, mk, dk, time.of.day, sat.lt, sat.ln, sat.ht, el, az, rng, -
                langle, head, sta.id$, sat.id$)

    'update time
    CALL time.update(0, delta.t.hr, tm, jd, jd(), tj, time.of.day, mk, dk, yk, -
                    h#, mo$, day$)

NEXT time

```

```

PRINT : PRINT "Press any key to continue."
FOR i% = 1 TO 10: c$ = INKEY$: NEXT
c$ = INPUT$(i)

-----
CASE "2"

CALL output4(0, y%, m%, d%, time.of.day, sat.lt, sat.ln, sat.ht, el, az, rng, -
    langle, head, sta.id$, sat.id$)

tm = 0
period = twopi / motion
ea = 0

time = begin.time
initialize:
CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
CALL culmnate(sta.z(), we, time, ea, coseta, elemu())
IF coseta < 0 THEN
    ea = ea + pi
    CALL culmnate(sta.z(), we, time, ea, coseta, elemu())
END IF
time = (ea - elem(2) * SIN(ea)) / motion + elemu(3)
dt.hr = (time - begin.time) / sixty
CALL time.update(0, dt.hr, tm, jd, jd(), tj, time.of.day, m%, d%, y%, h%, -
    mo$, day$)
IF time > begin.time + period THEN
    time = time - period
    CALL time.update(1, -period, tm, jd, jd(), tj, time.of.day, m%, d%, y%, -
        h%, mo$, day$)
    GOTO initialize
END IF

again:
FOR i% = 1 TO 2
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    CALL culmnate(sta.z(), we, time, ea, coseta, elemu())
    dt.min = (ea - elem(2) * SIN(ea)) / motion + elemu(3) - time
    time = time + dt.min
    CALL time.update(1, dt.min, tm, jd, jd(), tj, time.of.day, m%, d%, y%, -
        h%, mo$, day$)
NEXT
time.cul = time

CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)
IF el > 0 AND time >= begin.time THEN
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    ea = ea - .335
    FOR i% = 1 TO 2
        CALL rise.set(sta.z(), sta.R(), we, time, ea, elemu())
        dt.min = (ea - elem(2) * SIN(ea)) / motion + elemu(3) - time
        time = time + dt.min
        CALL time.update(1, dt.min, tm, jd, jd(), tj, time.of.day, m%, d%, -
            y%, h%, mo$, day$)
        CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    NEXT
    time.rise = time

CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)
CALL sat.sphere.coord(sat.x(), r, tj, tm, sat.dc, sat.ln, sat.ra)
CALL ground.track(r, sat.dc, sat.lt, sat.ht)
CALL sat.head(r, sat.x(), sat.xd(), head)
CALL jd.to.date(jd(), m%, d%, y%, h%, mo$, day$)
CALL output4(i, y%, m%, d%, time.of.day, sat.lt, sat.ln, sat.ht, el, -
    az, rng * ae, langle, head, sta.id$, sat.id$)

```

```

time.incr = dmod(time.cul - time.rise, delta.t)
time = time + time.incr
CALL time.update(i, time.incr, tm, jd, jd(), tj, time.of.day, m$, d$, -
y$, h$, mo$, day$)
CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)
CALL sat.sphere.coord(sat.x(), r, tj, tm, sat.dc, sat.ln, sat.ra)
CALL ground.track(r, sat.dc, sat.lt, sat.ht)
CALL sat.head(r, sat.x(), sat.xd(), head)
CALL jd.to.date(jd(), m$, d$, y$, h$, mo$, day$)
CALL output4(i, y$, m$, d$, time.of.day, sat.lt, sat.ln, sat.ht, el, -
az, rng * ae, langle, head, sta.id$, sat.id$)

DO WHILE el > 0
    time = time + delta.t
    CALL time.update(0, delta.t.hr, tm, jd, jd(), tj, time.of.day, m$, d$, -
y$, h$, mo$, day$)
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)
    IF el < 0 THEN
        EXIT DO
    END IF
    CALL sat.sphere.coord(sat.x(), r, tj, tm, sat.dc, sat.ln, sat.ra)
    CALL ground.track(r, sat.dc, sat.lt, sat.ht)
    CALL sat.head(r, sat.x(), sat.xd(), head)
    CALL jd.to.date(jd(), m$, d$, y$, h$, mo$, day$)
    CALL output4(i, y$, m$, d$, time.of.day, sat.lt, sat.ln, sat.ht, el, -
az, rng * ae, langle, head, sta.id$, sat.id$)
LOOP

save.time = time
FOR i% = 1 TO 2
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    CALL rise.set(sta.z(), sta.R(), we, time, ea, elemu())
    dt.min = (ea - elem(2) * SIN(ea)) / motion + elemu(3) - time
    time = time + dt.min
    CALL time.update(1, dt.min, tm, jd, jd(), tj, time.of.day, m$, d$, y$, -
h$, mo$, day$)
NEXT
    CALL pos.update(2, elem(), time, sat.x(), sat.xd(), elemu(), r, ea)
    CALL sta.sat(sta.x(), sat.x(), rng, az, el, langle)
    CALL sat.sphere.coord(sat.x(), r, tj, tm, sat.dc, sat.ln, sat.ra)
    CALL ground.track(r, sat.dc, sat.lt, sat.ht)
    CALL sat.head(r, sat.x(), sat.xd(), head)
    CALL jd.to.date(jd(), m$, d$, y$, h$, mo$, day$)
    CALL output4(i, y$, m$, d$, time.of.day, sat.lt, sat.ln, sat.ht, el, -
az, rng * ae, langle, head, sta.id$, sat.id$)
PRINT

END IF
time = time.cul + period
dt.hr = (time - begin.time) / sixty - tm
CALL time.update(0, dt.hr, tm, jd, jd(), tj, time.of.day, m$, d$, y$, h$, -
mo$, day$)

IF time < end.time THEN GOTO again
'-----
END SELECT
RETURN

input.sim.times:
CLS
PRINT "Simulation-Time Data:"
PRINT

PRINT "Starting date (yyyy.mmdd) ? "; startdate$

```

```

LOCATE 3, 29: INPUT v$: IF v$ = "" THEN v$ = startdate$
startdate$ = v$: CALL yrmaday(v$, yr&, mo&, dy&)

PRINT "Starting time (hh.mmss)      ? "; starttime$
LOCATE 4, 29: INPUT v$: IF v$ = "" THEN v$ = starttime$
starttime$ = v$: starttime = decimals(v$)
CALL julian.day.number(mo&, dy&, yr&, starttime, jdstart(), startdate, _
tjstart)
jd(1) = jdstart(1)
jd(2) = jdstart(2)
jd = jd(1) + jd(2)
time.of.day = starttime

PRINT "Ending date (yyyy.mmdd)      ? "; enddate$
LOCATE 5, 29: INPUT v$: IF v$ = "" THEN v$ = enddate$
enddate$ = v$: CALL yrmaday(v$, yr&, mo&, dy&)

PRINT "Ending time (hh.mmss)      ? "; endtime$
LOCATE 6, 29: INPUT v$: IF v$ = "" THEN v$ = endtime$
endtime$ = v$: endtime = decimals(v$)
CALL julian.day.number(mo&, dy&, yr&, endtime, jdend(), enddate, tjend)

PRINT "Time increment (minutes)      ? "; deltat$
LOCATE 7, 29: INPUT v$: IF v$ = "" THEN v$ = deltat$
deltat$ = v$: delta.t = VAL(v$): delta.t.hr = delta.t / sixty

LOCATE 9, 1: PRINT "Press any key to continue."
FOR i% = 1 TO 10: .3 - INKEY$: NEXT
c$ = INPUT$(1)

RETURN

'*****
FUNCTION acos# (x#)      ' 02-10-88      Rev. 02-24-88
' The arccosine function derived from the ATN function with no singularities.
' The angle is returned in the (0,pi) interval.

'Note that in relational comparisons, -1 is "true" and 0 is "false".

CONST pi = 3.141592653589793#, eps = 1D-33

IF ABS(x#) <= 1# THEN
  acos# = ATN(SQR(1# - x# * x#)) / (x# - eps * (x# = 0#)) - pi * (x# < 0#)
ELSE
  PRINT
  PRINT "Error in acos function. ABS(arg) > 1"
  PRINT "arg = "; x#
  PRINT
END IF
END FUNCTION

'*****
SUB apo.peri.gee (elem#(), elemus#()) ' 09-14-88.      Rev 09-14-88.
' Compute location and height of apogee and perigee above oblate earth.

' NOTE: (1) The output from this routine is approximately valid for the
'       current orbit only. In particular, the longitude is not corrected
'       for the earth's rotation during the current orbit. Also, the
'       ascending node and argument of perigee are not corrected for
'       their rates of change during the current orbit.
' (2) For nearly circular orbits, the height of the satellite will
'       probably get lower than the perigee value and higher than the
'       apogee value at locations other than the perigee and apogee
'       positions---this is an effect of the earth's oblateness.

```

```

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi

' spherical earth values:
apogee$ = elemu$(1) * (1$ - elem$(2))
perigee$ = elemu$(1) * (1$ + elem$(2))

' longitudes and declinations:
sincl$ = SIN(elem$(4))
speri$ = SIN(elem$(6))
apo.dcg = asin$(speri$ * sincl$)
apo.lnt = elem$(5) + atan2$(COS(elem$(4)) * speri$, COS(elem$(6)))
apo.lnt = dmod$(apo.lnt, twopi)
IF apo.lnt$ > pi THEN apo.lnt$ = apo.lnt$ - twopi
    ' next are approximations should update by peri- & nodal-rates
    ' for 1/2 revolution:
peri.dcg = -apo.dcg
peri.lnt = dmod(apo.lnt$ + pi, twopi)
IF peri.lnt$ > pi THEN peri.lnt$ = peri.lnt$ - twopi

CALL ground.track(apogee$, apo.dcg, apo.lnt$, apo.ht$)
CALL ground.track(perigee$, peri.dcg, peri.lnt$, peri.ht$)

PRINT
PRINT "Apogee:"
PRINT USING "# #####.##"; "Height      = "; apo.ht$; " km. "
PRINT USING "# #####.##"; "Latitude   = "; apo.lnt$ / rad
PRINT USING "# #####.##"; "Longitude = "; dmod(apo.lnt$ / rad, 360$)
PRINT
PRINT "Perigee:"
PRINT USING "# #####.##"; "Height      = "; peri.ht$; " km. "
PRINT USING "# #####.##"; "Latitude   = "; peri.lnt$ / rad
PRINT USING "# #####.##"; "Longitude = "; dmod(peri.lnt$ / rad, 360$)
PRINT

IF apo.ht$ < 0 THEN
    PRINT "WARNING: Satellite is suborbital and will impact earth."
    PRINT "Do you want to continue? Yes or No."
    PRINT
    WHILE c$ <> "y" AND c$ <> "n"
        c$ = LCASE$(INPUT$(1))
    WEND
    IF c$ = "n" THEN END
END IF
END SUB

*****FUNCTION asin$(x$)      ' 02-10-88  Rev. 02-24-88
' The arcsine function derived from the ATN function with no singularities.
' The angle is returned in the (-pi,pi) interval.

'Note that in relational comparisons, -1 is "true" and 0 is "false".

CONST eps = 1D-33

IF ABS(x$) <= 1$ THEN
    asin$ = ATN(x$ / (SQR(1$ - x$ * x$) - eps * (ABS(x$) = 1$)))
ELSE
    PRINT
    PRINT "Error in asin function. ABS(arg) > 1"
    PRINT "arg = "; x$
    PRINT
END IF
END FUNCTION

*****FUNCTION atan2$(y$, x$)      ' 02-10-88  Rev. 02-24-88

```

```

' The two argument quadrant determining arctangent function.
' The angle is returned in the (-pi,pi) interval.

' Note that in relational comparisons, -1 is "true" and 0 is "false".

CONST pi = 3.141592653589793$, eps = 1D-33

atan2$ = ATN(y$ / (x$ - eps * (x$ = 0$))) - pi * (x$ < 0$) * (SGN(y$) -
- (y$ = 0$))

END FUNCTION

*****SUB culmuate (zv$(), we$, t0$, ea$, coseta$, elemu$())
***** Compute eccentric anomaly for the time of culmination of a satellite
' 01-30-89. Rev. 02-02-89 @ 1000.

' NOTE: This particular form of the culminate function was written from the
' development of 30 Jan 89. It assumes a non-rotating earth so that the
' station longitude is fixed. Rotation is accomplished by subtracting the
' accumulated rotation from the right ascension of the ascending node of
' the satellite. This is the method used to compute the ephemeris and is
' also equivalent to the method used by NAVSPASUR.

' lambda = station geodetic longitude
' phi = station geodetic latitude
' we = earth's rotation (sidereal rate of change)
' t0 = time the elements, elemu(), were last updated
' capt = T -- time of latest perifocal passage
' mot = mean motion
' ea = eccentric anomaly E
' ecc = eccentricity e
' amjr = semimajor axis (cancels out in all formulas, so not included)

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi

ecc$ = elemu$(2)
capt$ = elemu$(3)
incl$ = elemu$(4)
node$ = elemu$(5)
peri$ = elemu$(6)
mot$ = elemu$(7)

DIM pv$(3), pvd$(3), qv$(3), qvd$(3), qvdd$(3)
DIM rvec$(3), rvecd$(3), rvecdd$(3)
' zv = Z-vector, unit normal at the station
' pv = P-vector, qv = Q-vector (first two columns of the Euler matrix)
' rvec = R-vector (satellite position), rvecd = dR/dE, rvecdd = d2R/dE2
' rv = r---magnitude of the R-vector, rvd = dr/dE, rvdd = d2r/dE2
' gamma = node - accrued rotational motion

cosw$ = COS(peri$)
sinw$ = SIN(peri$)
cosi$ = COS(incl$)
sini$ = SIN(incl$)

corr$ = 1
WHILE ABS(corr$) > .0001
  cosea$ = COS(ea$)
  ecosea$ = ecc$ * cosea$
  sinea$ = SIN(ea$)
  esinea$ = ecc$ * sinea$

  gammaw$ = node$ - we$ * ((ea$ - esinea$) / mot$ + capt$ - t0$)
  gammadd$ = -we$ * (1$ - ecosea$) / mot$
```

```

gammadd$ = -we$ * esinea$ / mots$
cosg$ = COS(gamma$)
sing$ = SIN(gamma$)

rtecc$ = SQR(1$ - ecc$ * ecc$)

rv$ = 1$ - ecosea$
rvd$ = esinea$
rvdd$ = ecosea$

xw$ = cosea$ - ecc$
xwd$ = -sinea$
xwdd$ = -cosea$
yw$ = rtecc$ * sinea$
ywd$ = rtecc$ * cosea$
ywdd$ = -yw$

pv$(1) = cosw$ * cosg$ - sinw$ * sing$ * cosi$
pv$(2) = cosw$ * sing$ + sinw$ * cosg$ * cosi$
pv$(3) = sinw$ * sini$
qv$(1) = -sinw$ * cosg$ - cosw$ * sing$ * cosi$
qv$(2) = -sinw$ * sing$ + cosw$ * cosg$ * cosi$
qv$(3) = cosw$ * sini$

pvd$(1) = -pv$(2) * gammadd$
pvd$(2) = pv$(1) * gammadd$
pvd$(3) = 0$
qvd$(1) = -qv$(2) * gammadd$
qvd$(2) = qv$(1) * gammadd$
pvd$(3) = 0

pvdd$(1) = -pvd$(2) * gammad$ - pv$(2) * gammadd$
pvdd$(2) = pvd$(1) * gammad$ + pv$(2) * gammadd$
pvdd$(3) = 0
qvdd$(1) = -qvd$(2) * gammad$ - qv$(2) * gammadd$
qvdd$(2) = qvd$(1) * gammad$ + qv$(2) * gammadd$
qvdd$(3) = 0

FOR i% = 1 TO 3
    rvec$(i%) = xw$ * pv$(i%) + yw$ * qv$(i%)
    rvecd$(i%) = xwd$ * pv$(i%) + xw$ * pvd$(i%) + ywd$ * qvd$(i%) -
                 + yw$ * qvd$(i%)
    temp$ = xwdd$ * pv$(i%) + 2$ * xwd$ * pvd$(i%) + xw$ * pvdd$(i%)
    rvecdds$(i%) = temp$ + ywdd$ * qv$(i%) + 2$ * ywd$ * qvd$(i%) -
                 + yw$ * qvdd$(i%)
NEXT

coseta$ = dot$(zv$(), rvec$()) / rv$
fe$ = dot$(zv$(), rvecd$()) - rvd$ * coseta$

rrd$ = rvd$ / rv$
fed$ = dot$(zv$(), rvecdds$()) + coseta$ * (rrd$ - rvdd$)
fed$ = fed$ - rrd$ * dot$(zv$(), rvecd$())

corr$ = fe$ / fed$
IF corr$ > pi THEN
    ea$ = ea$ + twopi
ELSE
    ea$ = ea$ - corr$
    IF coseta$ < 0 THEN ea$ = ea$ + pi
END IF

WEND
END SUB

```

```

*****
FUNCTION decimal$ (v$)
' Convert ddd.mmssfff or hh.mmssfff to decimal degrees or decimal hours.

ix = INSTR(v$, ".")
IF ix = 0 THEN
    decimal$ = VAL(v$)
ELSE
    x$ = VAL(LEFT$(v$, ix))
    sn = 0
    IF x$ < 0$ THEN
        sn = -1
        x$ = -x$
    END IF
    w$ = v$ + "0000"
    y$ = VAL(MID$(w$, ix + 1, 2))
    z$ = VAL(MID$(w$, ix + 3, 2)) + "." + RIGHT$(w$, LEN(w$) - ix - 4))
    x$ = (z$ / 60$ + y$) / 60$ + x$
    IF sn THEN x$ = -x$
    decimal$ = x$
END IF
END FUNCTION

*****
FUNCTION imod$ (x$, m$) ' 02-10-88 Rev. 02-24-88
' Provides x MOD m for real-valued functions.

IF m$ <> 0$ THEN
    dmod$ = x$ - m$ * INT(x$ / m$)
ELSE
    PRINT
    PRINT "Error in dmod function. Modulus cannot be zero."
    PRINT
END IF
END FUNCTION

*****
FUNCTION dot$ (a$(), b$())
' Dot product of two vectors.

dot$ = a$(1) * b$(1) + a$(2) * b$(2) + a$(3) * b$(3)

END FUNCTION

*****
SUB euler (peri$, node$, incl$, psn$(), vel$())
' 09-05-88. Rev. 01-16-89.

'Compute the first two columns (p & q) of the Euler matrix.
'Rotate in-plane position and velocity components to rectangular equatorial
' coordinates.
'psn() and vel() are both input and output vectors. It is assumed that the
'third component of both psn() and vel() is zero on input.

cw$ = COS(peri$)
sw$ = SIN(peri$)
cn$ = COS(node$)
sn$ = SIN(node$)
ci$ = COS(incl$)
si$ = SIN(incl$)
px$ = cw$ * cn$ - sw$ * sn$ * ci$
py$ = cw$ * sn$ + sw$ * cn$ * ci$
pz$ = sw$ * si$
qx$ = -sv$ * cn$ - cw$ * sn$ * ci$
qy$ = -sv$ * sn$ + cw$ * cn$ * ci$
qz$ = cw$ * si$
```

```

t1# = psn$(1)
t2# = psn$(2)
psn$(1) = px# * t1# + qx# * t2#
psn$(2) = py# * t1# + qy# * t2#
psn$(3) = pz# * t1# + qz# * t2#
t1# = vel$(1)
t2# = vel$(2)
vel$(1) = px# * t1# + qx# * t2#
vel$(2) = py# * t1# + qy# * t2#
vel$(3) = pz# * t1# + qz# * t2#

END SUB

'*****
FUNCTION gmst# (tu#, tm#) ' 03-03-88. Rev 03-03-88.
' Compute Greenwich mean sidereal time.
' tu# = number of centuries of 36525 days of universal time elapsed since
' 2000 Jan 1, 12h UT1 (JD 2451545 UT1).
' tm# = time of day
' The Astronomical Almanac, 1984, pp S13-S15.
' Almanac for Computers 1988, pp B2-B3.

CONST pi = 3.141592653589793#, rad = pi / 180#
CONST c0 = 24110.54841#, c1 = 8640184.812866#, c2 = 9.310400000000001D-02
CONST c3 = -.0000062#, hourpersec = 1# / 3600#, meantoapp = -.00029#
CONST d0 = 125.004452#, d1 = -.0529538#, d2 = .002071#

t# = (((c3 * tu# + c2) * tu# + c1) * tu# + c0) * hourpersec + tm#' GMST
'The following two lines will convert GMST to GAST
'lunarnode# = (d2 * tu# + d1) * tu# + t0#
't# = t# + meantoapp * SIN(lunarnode# * rad)
gmst# = dmod#(t#, 24#)

END FUNCTION

'*****
SUB ground.track (r#, sat.dc#, sat.lt#, sat.ht#) '09-14-88. Rev 09-14-88.

CONST ae = 6378.14# ' earth's equatorial radius (km.)
CONST f1 = 1# / 298.257# ' earth's flattening factor
CONST e.sqr = (2# - f1) * f1 ' earth's ellipticity squared
CONST f2 = (1# - f1) * (1# - f1)

rkm# = r# * ae
ps# = sat.dc#
el: cp# = COS(ps#)
    rc# = ae * SQR((1# - e.sqr) / (1# - e.sqr * cp# * cp#))
    rcr# = rc# / rkm#
    sat.lt# = ATN(TAN(ps#) / f2) ' satellite latitude
    sn# = SIN(sat.lt# - ps#)
    hr# = SQR(1# - rcr# * rcr# * sn# * sn#) - rcr# * COS(sat.lt# - ps#)
    pn# = sat.dc# - asin(hr# * sn#)
    IF (ABS(pn# - ps#) > .0000001) THEN ps# = pn#: GOTO el
    sat.ht# = hr# * rkm# ' satellite height

END SUB

'*****
SUB hr.min.sec (x#, hr%, min%, sec%, n%)
' Convert decimal hours to: hh mm ss if n% = 0,
'                           hh mm ss.f if n% = 1,
'                           hh mm ss.ff if n% = 2,
'                           hh mm ss.fff if n% = 3.

CONST t0 = 1#, t1 = 10#, t2 = 100#, t3 = 1000#

```

```

CONST r0 = 1# / 7200#, r1 = r0 / t1, r2 = r1 / t2, r3 = r2 / t3

SELECT CASE n%
CASE 0
    round = r0
    trunc = t0
CASE 1
    round = r1
    trunc = t1
CASE 2
    round = r2
    trunc = t2
CASE 3
    round = r3
    trunc = t3
CASE ELSE
    PRINT
    PRINT "Error in hr.min.sec subroutine. Must have 0 <= n% <= 3."
    PRINT "n% = "; n%
    PRINT
    EXIT SUB
END SELECT

v# = ABS(x#) + round
hr% = INT(v#)
v# = (v# - hr%) * 60#
min% = INT(v#)
sec# = INT((60# * (v# - min%)) * trunc)) / trunc

END SUB

'*****
SUB jd.to.date (jd#(), m#, d#, y#, h#, mo$, day$) ' 02-24-88 Rev. 03-02-88
' Convert Julian Day Number to month number, day, year, hour, month name and
' day of the week. Limited to the Gregorian Calendar.
' Fliegel & VanFlandern, Comm. ACM, Vol. 11, No. 10, Oct. 1968, pg. 657.

CONST w$ = "MonTueWedThuFriSatSun"
CONST dd$ = "JanFebMarAprMayJunJulAugSepOctNovDec"

j4# = INT(jd#(1) + jd#(2) + .5)
h# = ((jd#(1) - j4#) + jd#(2) + .5) * 24#
y# = INT(j4# + 68569)
n# = INT(4 * y# / 146097)
l# = l# - INT((146097 * n# + 3) / 4)
y# = INT(4000 * (l# + 1) / 1461001)
l# = l# - INT(1461 * y# / 4) + 31
m# = INT(80 * l# / 2447)
d# = l# - INT(2447 * m# / 80)
l# = INT(m# / 11)
m# = m# + 2 - 12 * l#
y# = 100 * (n# - 49) + y# + l#

wn% = j4# - 7 * INT(j4# / 7) + 1
day$ = MID$(w$, 3 + wn% - 2, 3)
mo$ = MID$(dd$, 3 * m# - 2, 3)

END SUB

'*****
SUB julian.day.number (m#, d#, y#, ut#, jd#(), dj#, tj#)
' 02-25-88 Rev. 03-23-88
' Conversion of calendar date: m# = month, d# = day, y# = 4-digit year,
' and ut# = Universal Time, to Julian Day Number. Year must be between 1801
' and 2099.

```

```

' Almanac for Computers, 1988, pg. B-2.

' Note that jd$ is a subscripted double precision array. At any time, the
' Julian Day Number is given by jd$(1) + jd$(2).
' For ease of programming, the user may put the entire epoch in jd$(1) and set
' jd$(2) = 0$.
' For maximum accuracy, set jd$(1) = the most recent midnight at or before the
' epoch and set jd$(2) = the fractional part of a day elapsed between jd$(1)
' and the epoch. As an example, compute the number of days from epoch
' J2000.0 = JD 2451545.0 using the code:
'           days$ = (jd$(1) - 2451545.0$) + jd$(2)
' Note, particularly, the extra pair of parenthesis.
'
' dj$ = days and fraction from J2000.0. tj$ = julian centuries from J2000.0

```

```

CONST c1 = 1721013.5$, c2 = 190002.5$, jcent = 36525$

IF (1801 <= y$) AND (y$ <= 2099) THEN
    jd$(1) = 367 * y$ - INT(7 * (y$ + INT((m$ + 9) / 12)) / 4) -
        + INT((275 * m$) / 9) + d$ + c1 - .5$ * SGN(100 * y$ + m$ - c2) + .5$
    jd$(2) = ut$ / 24
    dj$ = (jd$(1) - 2451545$) + jd$(2)
    tj$ = dj$ / jcent
ELSE
    PRINT
    PRINT "Error in julian.day.number. Year is not between 1801 and 2099."
    PRINT "Year = "; y$
    PRINT
END IF
END SUB

```

```

*****SUB kepler (m$, ec$, ea$) ' 02-25-88. Rev 02-26-88

```

```

'High precision, non-iterative, solution to Kepler's equation for the ellipse.
'Accuracy is 10E-18 using all terms or 10E-15 if the last term is omitted.
'Although not as compact as the Newton-Raphson procedure, one square root, one
'cube root, and only two trigonometric functions are evaluated thus making this
'procedure extremely fast. Can be extended to the hyperbolic case - see Ref.
'Ref.: S. Mikkalo, Cel. Mech., Vol. 40, 1987, pp 329-334.

```

```

'm$ = mean anomaly, ec$ = eccentricity and ea$ = eccentric anomalous.

```

```

CONST pi = 3.141592653589793$, twopi = pi + pi, third = 1$ / 3$
CONST r3 = 1$ / 6$, r4 = 1$ / 24$, r5 = 1$ / 120$

IF ec$ >= 1$ OR ec$ < 0$ THEN
    PRINT
    PRINT "Error in Kepler. The eccentricity is not in the [0,1) interval."
    PRINT "Eccentricity = "; ec$
    PRINT
ELSE
    m$ = m$ - twopi * INT(m$ / twopi)' assure that m$ is
    IF m$ > pi THEN m$ = m$ - twopi' in the (-pi,pi) interval

    'Compute the initial approximation:
    g$ = 1 / (4 * ec$ + .5)
    a$ = (1 - ec$) * g$
    b$ = .5 * m$ * g$
    arg$ = ABS(b$) + SQR(b$ * b$ + a$ * a$ * a$)
    z$ = arg$ ^ third
    IF b$ < 0 THEN z$ = -z$
    s$ = z$ - a$ / z$
    ss = s$ - .078 * s$ ^ 5 / (1 + ec$)
    ea$ = m$ + ec$ * s$ * (3 - 4 * a$ * s$)' abs(dea/ea) <= 0.002

```

```

'Refine the approximation:
f2$ = ec$ * SIN(ea$)
f3$ = ec$ * COS(ea$)
f0$ = ea$ - f2$ - m$
f1$ = 1$ - f3$
f4$ = -f2$
f5$ = -f3$
'This next correction term is sufficient if ecc < 0.25
d$ = -f0$ / f1$
'Use more of the following terms to increase accuracy
d$ = -f0$ / (f1$ + .5$ * f2$ * d$)
d$ = -f0$ / (f1$ + d$ * (.5$ * f2$ + d$ * r3 * f3$))
d$ = -f0$ / (f1$ + d$ * (.5$ * f2$ + d$ * (r3 * f3$ + d$ * r4 * f4$)))
d$ = -f0$ / (f1$ + d$ * (.5$ * f2$ + d$ * (r3 * f3$ + d$ -
    * (r4 * f4$ + d$ * r5 * f5$))))
ea$ = ea$ + d$
END IF
END SUB

*****
SUB output4 (k%, yr%, mo%, da%, hr%, min%, sec%, lt, ln, ht, el, az, rng, -
    langle, head, sta.id$, sat.id$)

CONST pi = 3.141592653589793$, rad = pi / 180$, tp = pi + pi
c$ = CHR$(34)

SELECT CASE k%
CASE 0
    PRINT "                                     height      ";
    PRINT "           look"
    PRINT "year mo da hr mn sec   lat   long   (km)   elev   ";
    PRINT "azim  range angle  head"
    PRINT
CASE 1
    CALL hr.min.sec(time.of.day$, hr%, min%, sec$, 1)
    lt = lt / rad
    ln = dmod(ln / rad, 360$)
    la = langle
    hd = head / rad
    PRINT USING "#### #.## #.## #.## #.##"; yr%; mo%; da%; hr%; min%; sec;
    PRINT USING "#.## #.## #.## #.## #.##"; lt; ln; ht; el;
    PRINT USING "#.## #.## #.## #.## #.##"; az; rng; la; hd
    PRINT #3, USING "!&! !&! "; c$; sta.id$; c$; c$; sat.id$; c$;
    PRINT #3, USING "!###! ###! "; c$; yr%; c$; c$; mo%; c$;
    PRINT #3, USING "!##! ##! "; c$; da%; c$; c$; hr%; c$;
    PRINT #3, USING "!##! ##.##! "; c$; min%; c$; c$; sec; c$;
    PRINT #3, USING "#.## #.## #.## #.##! "; c$; lt; c$; c$; ln; c$;
    PRINT #3, USING "#.## #.## #.## #.##! "; c$; ht; c$; c$; el; c$;
    PRINT #3, USING "#.## #.## #.## #.##! "; c$; az; c$; c$; rng; c$;
    PRINT #3, USING "#.## #.## #.## #.##! "; c$; la; c$; c$; hd; c$
CASE ELSE
    PRINT
    PRINT "Error in output4 subroutine. Must have 0 <= k% <= 1."
    PRINT "k% = "; k%
END SELECT
END SUB

*****
SUB pos.update (1%, elem$(), time$, sat.x$(), sat.xd$(), elemu$(), r$, ea$) -
    STATIC
    ,
    09-07-88. Rev. 03-09-89.

    ' Case 1: Initialize orbital elements for time of epoch.
    ' Case 2: Update orbital elements and compute in-plane position & velocity.
    ' Simplest of NAVSPASUR methods. Accurate to 10-15 n.mi. within 20 days

```

```

' of epoch.

' elem(1) = amajor      semi-major axis
' elem(2) = ecc          eccentricity
' elem(3) = tzero        time of perifocal passage
' elem(4) = aincl        inclination (radians)
' elem(5) = anode        longitude of ascending node (radians)
' elem(6) = peri         argument of perifocus (radians)
' elem(7) = motion       mean motion (revolutions/day)
' elem(8) = manomaly     mean anomaly (radians)
' elem(9) = decay        1st decay constant (radians/minute^2)

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi
CONST c = .00081197385$ , 0.75*j2
CONST twothird = 2$ / 3$
CONST ae = 6378.14$           ' earth's equatorial radius (km.)
CONST GE = 398600.5$          ' geo. grav. const. (km)^3/(sec)^2
k = 60$ * SQR(GE / ae) / ae^(eru)^(3/2)/min
CONST we = 1.002737909350795$ * twopi / 1440$ sidereal period in radians/min

SELECT CASE 1%
CASE 1
    elem$(1) = (k / elem$(7)) ^ twothird
    a2$ = elem$(1) ^ 2
    ecc2$ = 1$ - elem$(2) ^ 2
    ci$ = COS(elem$(4))
    c2i$ = ci$ * ci$
    elem$(1) = elem$(1) * (1$ + c * (3$ * c2i$ - 1$) / -
                           (a2$ * ecc2$ ^ 1.5$)) ^ twothird
    elem$(3) = time$ - elem$(8) / elem$(7)
    aecc22$ = (elem$(1) * ecc2$) ^ 2
    peridot$ = c * (5$ * c2i$ - 1$) / aecc22$
    nodedot$ = -2$ * c * ci$ / aecc22$
    adot$ = -2$ * twothird * elem$(1) * m2$ / elem$(7)
    FOR i% = 1 TO 9
        elemu$(i%) = elem$(i%)
    NEXT
CASE 2
    ma.m0$ = (elem$(9) * time$ + elem$(7)) * time$
    ma$ = ma.m0$ + elem$(8)
    elemu$(6) = elem$(6) + peridot$ * ma.m0$
    elemu$(5) = elem$(5) + nodedot$ * ma.m0$ - we * time$
    elemu$(1) = elem$(1) + adot$ * time$
    CALL kepler(ma$, elem$(2), ea$)
        s0$ = SIN(ea$)
        c0$ = COS(ea$)
    elemu$(8) = ma$
    elemu$(3) = time$ - ma$ / elem$(7)

    ' in-plane position and velocity
    sat.x$(1) = elemu$(1) * (c0$ - elem$(2))
    sat.x$(2) = elemu$(1) * SQR(ecc2$) * s0$
    sat.x$(3) = 0$
    r$ = elemu$(1) * (1$ - elem$(2) * c0$)
    ed$ = elemu$(1) * elem$(7) / r$
    sat.xd$(1) = -elemu$(1) * ed$ * s0$
    sat.xd$(2) = elemu$(1) * ed$ * SQR(ecc2$) * c0$
    sat.xd$(3) = 0$

    ' equatorial rectangular coordinates
    CALL euler(elemu$(6), elemu$(5), elem$(4), sat.x(), sat.xd())

    ' correct velocity for earth's rotation
    sat.xd$(1) = sat.xd$(1) + we * sat.x(2)
    sat.xd$(2) = sat.xd$(2) - we * sat.x(1)

```

```

CASE ELSE
PRINT
PRINT "Error in pos.update. First argument must be 1 or 2 only."
PRINT "Argument = "; 1%
PRINT
END SELECT
END SUB

*****SUB rise.set (zv$( ), sta.R$( ), we$, t0$, ea$, elemu$( ))
' Compute eccentric anomaly for the time of rise or set of a satellite
' 02-04-89. Rev. 02-08-89 @ 1700.

' NOTE: To use this routine, the time of culmination MUST be computed first,
' The entering ea$ argument should be ea.cul$ -/+ 0.2 radians (about 11
' degrees) for rise/set.

' See the introductory comments in SUB culminate for nomenclature and
' notation.

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi

amaj$ = elemu$(1)
ecc$ = elemu$(2)
capt$ = elemu$(3)
incl$ = elemu$(4)
node$ = elemu$(5)
peri$ = elemu$(6)
mot$ = elemu$(7)

DIM pv$(3), pvd$(3), qv$(3), qvd$(3), rvecd$(3), rho$(3)

cosw$ = COS(peri$)
sinw$ = SIN(peri$)
cosi$ = COS(incl$)
sini$ = SIN(incl$)

corr$ = 1
DO
  cosea$ = COS(ea$)
  ecosea$ = ecc$ * cosea$
  sinea$ = SIN(ea$)
  esinea$ = ecc$ * sinea$

  gamma$ = node$ - we$ * ((ea$ - esinea$) / mot$ + capt$ - t0$)
  gammad$ = -we$ * (1$ - ecosea$) / mot$

  cosg$ = COS(gamma$)
  sing$ = SIN(gamma$)

  rtecc$ = amaj$ * SQR(1$ - ecc$ * ecc$)

  xv$ = amaj$ * (cosea$ - ecc$)
  xvd$ = -amaj$ * sinea$
  yv$ = rtecc$ * sinea$
  yvd$ = rtecc$ * cosea$

  pv$(1) = cosw$ * cosg$ - sinw$ * sing$ * cosi$
  pv$(2) = cosw$ * sing$ + sinw$ * cosg$ * cosi$
  pv$(3) = sinw$ * sini$

  qv$(1) = -sinw$ * cosg$ - cosw$ * sing$ * cosi$
  qv$(2) = -inw$ * sing$ + cosw$ * cosg$ * cosi$
  qv$(3) = cosw$ * sini

```

```

pvd$(1) = -pv$(2) * gammads
pvd$(2) = pv$(1) * gammads
pvd$(3) = 0$
qvd$(1) = -qv$(2) * gammads
qvd$(2) = qv$(1) * gammads
qvd$(3) = 0$

FOR i% = 1 TO 3
    rho$(i%) = xv$ * pvs(i%) + yv$ * qvs(i%) - sta.R$(i%)
    rvecd$(i%) = xvd$ * pvs(i%) + xv$ * pvd$(i%) + yvd$ * qvs(i%) -
                  + yv$ * qvd$(i%)
NEXT

fe$ = dot$(zv$(), rho$())
fed$ = dot$(zv$(), rvecd$())

corr$ = fe$ / fed$
ea$ = ea$ - corr$

LOOP UNTIL ABS(corr$) < .00001$
END SUB

'*****
SUB sat.head (r$, sat.x$(), sat.xd$(), head$) ' 01-18-89. Rev. 03-12-89.
'satellite heading

CONST pi = 3.141592653589793$, twopi = pi + pi

shead$ = (sat.x$(1) * sat.xd$(2) - sat.x$(2) * sat.xd$(1)) / r$
chead$ = sat.xd$(3) - (sat.x$(3) * dot$(sat.x$(), sat.xd$())) / r$ ^ 2
head$ = atan2$(shead$, chead$)
IF head$ < 0$ THEN head$ = head$ + twopi

END SUB

'*****
SUB sat.sphere.coord (sat.x$, r, tj, tm, sat.dc, sat.ln, sat.ra)
'compute spherical coordinates

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi

sat.dc = asin(sat.x$(3) / r)           ' satellite declination
sat.ln = atan2(sat.x$(2), sat.x$(1))   ' satellite longitude
ru = 15$ * gmst(tj, tm) * rad          ' Greenwich mean sidereal time
sat.ra = dmod(sat.ln + ru, twopi)       ' satellite right ascension

END SUB

'*****
SUB sta.p.coord (lt$, ln$, ht$, x$()) ' 08-06-88. Rev 02-04-89.
' convert station, lat, long & height (ft.) from geographic
' to geocentric rectangular - x(.)

CONST nmi.ft = .3048$ / 1852$ ' n.mi./foot
CONST km.ft = .3048$ / 1000$ ' km./foot
CONST ae = 6378.148           ' earth's equatorial radius (km.)
CONST f1 = 18 / 298.257$      ' earth's flattening factor
CONST e.sqr = (28 - f1) * f1  ' earth's ellipticity squared

ht.eru$ = ht$ * km.ft / ae
slt$ = SIN(lt$)
clt$ = COS(lt$)
sln$ = SIN(ln$)
cln$ = COS(ln$)
n$ = 18 / SQR(18 - e.sqr * slt * slt)
g1$ = n$ + ht.eru$

```

```

g2$ = n$ * (1$ - e.sqr) + ht.eru$

'radius vector for flattened earth
x$(1, 1) = g1$ * clt$ * cln$
x$(2, 1) = g1$ * clt$ * sln$
x$(3, 1) = g2$ * slt$

'Spherical earth unit vectors:
'1) local perpendicular vector
x$(1, 2) = clt$ * cln$
x$(2, 2) = clt$ * sln$
x$(3, 2) = slt$

'2) east vector
x$(1, 3) = -sln$
x$(2, 3) = cln$
x$(3, 3) = 0$

'3) north vector
x$(1, 4) = -slt$ * cln$
x$(2, 4) = -slt$ * sln$
x$(3, 4) = clt$

END SUB

*****SUB sta.sat (sta.x$(), sat.x$(), rng$, az$, el$, langle$)
' 08-06-88. Rev. 08-31-88.

'compute station-satellite relations. Range, azimuth, elevation and satellite
' "look-angle" (angle between satellite sub point and station).

CONST pi = 3.141592653589793$, rad = pi / 180$, twopi = pi + pi

'rho() = sat coords minus sta coords. rhodots are the rho components projected
'in the station local north-east-radial system,
DIM rho$(3), rhodots$(3)

rng$ = 0$
FOR i% = 1 TO 3
    r$ = sat.x$(i%) - sta.x$(i%, 1)
    rho$(i%) = r$
    rng$ = rng$ + r$ * r$
NEXT
rng$ = SQR(rng$)
FOR i% = 1 TO 3
    rho$(i%) = rho$(i%) / rng$
NEXT

FOR i% = 1 TO 3
    a$ = 0$
    FOR j% = 1 TO 3
        a$ = a$ + rho$(j%) * sta.x$(j%, i% + 1)
    NEXT
    rhodots$(i%) = a$
NEXT

az$ = atan2$(rhodots$(2), rhodots$(3)) / rad
IF az$ < 0$ THEN az$ = az$ + 360$

a$ = rhodots$(1)
el$ = ATN(a$ / SQR(1$ - a$ * a$)) / rad

a$ = 0$
b$ = 0$
FOR i% = 1 TO 3

```

```

c$ = sat.x$(i%)
a$ = a$ + rho$(i%) * c$
b$ = b$ + c$ * c$
NEXT
langle$ = acos$(a$ / SQR(b$)) / rad

END SUB

'*****
SUB time.update (k%, delta.t$, tm$, jd$, jd$(), tj$, time.of.day, mk, dk, _
y$, h$, mo$, day$)

CONST twentyfour = 24$, sixty = 60$, jul.cent = 36525$

t$ = delta.t$                                'delta.t$ in hours if k% = 0, t$ in hours.
IF k% > 0 THEN t$ = t$ / sixty      'delta.t$ in minutes if k% > 0, t$ in hours.
tm$ = tm$ + t$
jd$(2) = jd$(2) + t$ / twentyfour
jd$ = jd$(1) + jd$(2)
tj$ = tj$ + t$ / twentyfour / jul.cent
time.of.day = time.of.day + t$
IF time.of.day >= twentyfour THEN
    time.of.day = time.of.day - twentyfour
    jd$(1) = jd$(1) + 1$: jd$(2) = jd$(2) - 1$
    CALL jd.to.date(jd$(()), mk, dk, y$, h$, mo$, day$)
END IF
IF time.of.day < 0$ THEN
    time.of.day = time.of.day + twentyfour
    jd$(1) = jd$(1) - 1$: jd$(2) = jd$(2) + 1$
    CALL jd.to.date(jd$(()), mk, dk, y$, h$, mo$, day$)
END IF

END SUB

'*****
SUB yrmoday (v$, yr$, mo$, dy$) ' 03-21-88. Rev 03-21-88.
' Convert yyyy.mmdd to yr$=yyyy, mo$=mm, and dy$=dd

IF INSTR(v$, ".") <> 5 OR LEN(v$) <> 9 THEN
    PRINT
    PRINT "Error in yrmoday. v$ must be of the form yyyy.mmdd"
    PRINT "v$ = "; v$
    PRINT
ELSE
    yr$ = VAL(LEFT$(v$, 4))
    mo$ = VAL(MID$(v$, 6, 2))
    dy$ = VAL(RIGHT$(v$, 2))
END IF
END SUB

```

APPENDIX E. THE HIGH ACCURACY KINEMATICS ROUTINE.

This appendix contains a listing of the alternate pos.update routine. It is a BASIC version of the highly accurate PPT2 subroutine provided by NAVSPASUR [Ref. 1] with their SHOWALL program. It is believed that it has been accurately translated, however any errors are solely those of the author. Known discrepancies are the difference in the WGS-72 constants used in SHOWALL and the IAU 1976 constants used here [see section III. SOURCES, in this report for details].

```

*****SUB pos.update (ind%, elem$( ), time$, sat.x$( ), sat.xd$( ), elemu$( ), -
r$, ea$) STATIC Rev. 09-26-89 & 0900

' Case 1: Initialize orbital elements for time of epoch.
' Case 2: Update orbital elements and compute in-plane position & velocity.

' Comments from original FORTRAN version:
' implicit real*8(a-h,o-z)
'*** model reference-Astronomical Journal 64, No 1274, Nov., '59
'*** solution of the problem of artificial satellite theory
'*** with out drag...Dirk Brouwer.
' 840329 confirmed use of w=u x v vice w=u x vel
' 840329 inserted cdh,sdh, cdt,sdt re osc vice mean els at t
' for use only with dcext2,dc1,dcia
' frame for u,v,w is the rotated inertial w/o coriolis
' 830127 new partials re (a,x,y,z,p,q)
' 830127 improved (polynomial) model for singularity in Brouwer

DIM a(25)
DIM f(25), osc(10), kf(10), cf(10), bs(3, 4), u(3), v(3), w(3), vel(3)
DIM ar(6, 8), br(3, 6), cr(3, 6), dv(3)

'
'*** The commented beta value is the WGS-72 value,however using this
'*** value, one obtains a value for a(9) different then what we had
'*** been using, thus causing a problem with time.therefore to retain
'*** previous value of a(9),one solves for beta using the old value
'*** of a(9).
'*** betai=398600.5d0
'*** beta1 = 398597.62579588$, erkm = 6378.135$, flat = 298.26$
' k = .0743669161$

' IAU 1976:
CONST twopi = 6.283185307179586$
CONST we = 1.002737909350795$ * twopi / 1440$'sidereal period in radians/min
CONST ae = 6378.14$, erkm = ae           ' earth's equatorial radius (km.)
CONST betai = 398600.5$                  ' geo. grav. const. (km)^3/(sec)^2
k = 60$ * SQR(betai / ae) / ae        '(eru)^(3/2)/min
CONST flat = 1$ / 298.257$              ' earth's flattening factor

'      k-terms      [Note: These may or may not agree with IAU 1976]
CONST c20 = -.0004841605$, c30 = .00000095958$
CONST c40 = .00000055199$, c50 = .000000065875$
'
CONST twotrd = .666666666667$, fortrd = 1.333333333333333$
CONST tentrd = 3.3333333333333$, ar = 48 * 0$

'IF ind% <> 1 THEN GOTO secondcase
SELECT CASE ind%
CASE 1

kz% = 1
a(1) = 0$' [Note: = 85-01-01 in SHOWALL. Set to zero here.]
```

```

a(3) = -.5# * c20 * SQR(5#)
a(4) = c30 * SQR(7#)
a(5) = .375 * c40 * SQR(9#)
a(6) = c50 * SQR(11#)
,
two pi
a(7) = twopi
a(16) = 1# / a(7)
'    hergs/day , secs/herg, mins/herg
a(9) = erkm * SQR(erkm / beta1)
a(8) = 86400# / a(9)
a(17) = 1440# / a(8)
'    we(rad/day), we- 2 pi, we(rad/herg)
a(11) = 6.3003880987#
a(10) = a(11) - a(7)
a(12) = a(11) / a(8)
'    earth flattening
a(13) = (2# - 1# / flat) / flat
'    sm/er, km/er, nm/er
a(20) = erkm / 1.609344#
a(21) = erkm
a(22) = erkm / 1.852#
'    deg/rad
a(23) = 360# / twopi
'    range rate/er/herg to cycles/second - conversion
a(24) = a(21) * 216980000# / (a(9) * 299792.5#)
'    fence plane displac from earth center
a(25) = .0031#

' Patch to interface PPT2 and SATSTA variables:
f(1) = elem(8)      'mean anomaly
f(2) = elem(7) * a(17) 'mean motion
f(3) = elem(9) * a(17) ^ 2'first decay constant
f(4) = 0#            'second decay constant
f(5) = elem(2)       'eccentricity
f(6) = elem(6)       'arg of perigee
f(7) = elem(5)       'node
f(8) = COS(elem(4))  'cosine inclination
f(9) = 0#            'epoch in PPT2, time from epoch here (? but check ?)

' Used only once to "recover" the remaining elements
h1 = 0#
'*** f(8) is cosine of inclination
t9 = f(8) * f(8)
t1 = 1# - 5# * t9
'    t2=(1.0d0-exp(-100.0d0*t1^2))/t1
'    revised handling of divisor per navspasur c2t memo nov '83
beta = 100# / 2# ^ 11
p3 = beta * t1 ^ 2
p1 = EXP(-p3)
p2 = 1# + p1
p1ex = p1 ^ 2
p4 = 1#
p3ex = -.5# * p3
FOR n% = 2 TO 13
    IF n% <= 11 THEN p2 = p2 * (1# + p1ex)
    p1ex = p1ex * p1ex
    p4 = p4 + p3ex
    p3ex = -p3 * p3ex / (n% + 1)
NEXT
t2 = p2 * p4 * beta * t1
cio2 = SQR(.5# + .5# * f(8))
sio2 = SQR(.5# - .5# * f(8))
t12 = 3# * t9 - 1#
t13 = t9 * t9
t14 = t2 * t13
t15 = 1# - t9

```

```

'*** f(5) is eccentricity
esq0 = f(5) * f(5)
eta20 = 1# - esq0
eta0 = SQR(eta20)
t6 = eta0 * eta20
t7 = eta20 * eta20
t8 = eta0 + 1# / (1# + eta0)
f(13) = f(2) ^ (-twotrd)
elem(1) = f(13)
'*** f(2) is basically mean motion but also contains the
'*** secular term in mean anomaly. therefore f(13) is (almost) the
'*** semi major axis.

'*** This loop is to remove the contribution of the additional
'*** term in mean motion for improving values of semi-major axis
'*** and seculars.
'*** a(3) to a(6) are 'k' terms wgs 72
FOR i% = 1 TO 5
    fs13 = f(13) * f(13) * t7
    fs11 = a(3) / fs13
    fs13 = a(5) / (fs13 * fs13)
    tt = 1# + 1.5# * fs11 * eta0 * t12
    uu = -15# + 16# * eta0 + 25# * eta20
    uu = uu + (30# - 96# * eta0 - 90# * eta20) * t9
    uu = uu + (105# + 144# * eta0 + 25# * eta20) * t13
    tt = tt + .09375# * fs11 ^ 2 * eta0 * uu
    tt = tt + .9375# * fs13 * eta0 * esq0 * (3# - 30# * t9 + 35# * t13)
    f(13) = (tt / f(2)) ^ twotrd
NEXT
'*** f(11) will be rate of change of argument of perigee
uu = -35# + 24# * eta0 + 25# * eta20
uu = uu + (90# - 192# * eta0 - 126# * eta20) * t9
uu = uu + (385# + 360# * eta0 + 45# * eta20) * t13
tt = -1.5# * fs11 * t1 + .09375# * fs11 ^ 2 * uu
uu = 21# - 9# * eta20 + (-270# + 126# * eta20) * t9
uu = uu + (385# - 189# * eta20) * t13
f(11) = tt + .3125# * fs13 * uu
'*** f(12) will be rate of change of right ascension
uu = -5# + 12# * eta0 + 9# * eta20
uu = uu - (35# + 36# * eta0 + 5# * eta20) * t9
tt = -3# * fs11 * f(8) + .375# * fs11 ^ 2 * f(8) * uu
f(12) = tt + 1.25# * fs13 * f(8) * (5# - 3# * eta20) * (3# - 7# * t9)
'*** f(15)=sine inclination,f(14)is a dot,f(16) is eccen.dot
f(15) = SQR(t15)
f(14) = -fortrd * f(3) * f(13) / f(2)
f(16) = f(14) * f(5) * eta20 / f(13)
q1 = f(2) * f(13) ^ 1.5#
f(11) = f(11) / q1
f(12) = f(12) / q1
fs12 = a(4) / (a(3) * f(13) * eta20)
fs13 = fs13 / fs11 * tentrd
fs14 = a(6) / (a(3) * f(13) ^ 3 * eta20 * t7)
q1 = .125# * (fs11 * (1# - 11# * t9 - 40# * t14) - fs13 -
        * (1# - 3# * t9 - 8# * t14))
p5 = 1# + t2 * (8# * t9 + 20# * t14)
p2 = 1# + 2# * p5
q2 = .125# * esq0 * f(8) * (fs11 * (1# + 10# * p5) - fs13 * p2)
p2 = .46875# * p2 * f(5) * f(8) * f(15) * (4# + 3# * esq0) * fs14
p3 = fs14 * (1# - 9# * t9 - 24# * t14)
q5 = .25# * (fs12 + .3125# * (4# + 3# * esq0) * p3)
p3 = .15625# * f(5) * f(15) * p3
p4 = .030381944# * f(5) * fs14 * (1# - 5# * t9 - 16# * t14)
p5 = .060763889# * f(5) * esq0 * f(8) * f(15) * fs14 * (1# + 4# * p5)
vl1 = f(5) * eta20 * q1
vlh1 = -f(15) * q2
tt = (t6 - 1#) * q1 - q2 + 25# * esq0 * t14 * t9 * t2 * (fs11 - .2# * fs13)

```

```

vls1 = tt - .0625$ * esq0 * (f11 * (1$ - 33$ * t9 - 200$ * t14) -
    - fs13 * (1$ - 9$ * t9 - 40$ * t14))
vle2 = eta20 * f(15) * q5
vlh2i = f(5) * f(8) * q5 + f(15) * p2
vls2 = f(5) * f(15) * (t8 + f(8)) / (1$ + f(8)) * q5 -
    + (11$ + 3$ * esq0 - 3$ * t6) * p3 + (1$ - f(8)) * p2
vle3 = -3$ * f(5) * eta20 * f(15) * p4
vh3i = -esq0 * f(8) * p4 - f(15) * p5
vls3 = f(15) * (3$ * t6 - 3$ - 2$ * esq0 - esq0 * f(8) / (1$ + f(8))) * p4 -
    - (1$ - f(8)) * p5
vll2 = vle2 + 3$ * f(5) * eta20 * p3
' precomps for the partials
'*** f(6) is argument of perigee, f(7) is right ascension
sinh0 = SIN(f(7))
cosh0 = COS(f(7))
sint0 = SIN(f(6) + f(7))
cost0 = COS(f(6) + f(7))
' kepler's law
dadm = -twotrd * f(13) / f(2)
' variations thru the seculars
secul = a(3) / (f(13) * eta20) ^ 2
dgddm = 3$ * secul * t1 / f(13) * dadm
dgdde = -6$ * secul * t1 * f(5) / eta20
dgddi = -15$ * secul * f(8) * f(15)
dhddm = 6$ * secul * f(8) / f(13) * dadm
dhdde = -12$ * secul * f(8) * f(5) / eta20
dhddi = 3$ * secul * f(15)
dtddx = (dgdde + dhdde) * cost0
dtddy = (dgdde + dhdde) * sint0
dtddp = (dgddi + dhddi) * 2$ * cosh0 / cio2
dtddq = (dgddi + dhddi) * 2$ * sinh0 / cio2
dhddx = dhdde * cost0
dhddy = dhdde * sint0
dhddp = dhddi * 2$ * cosh0 / cio2
dhddq = dhddi * 2$ * sinh0 / cio2
' variations thru m2
daddm1 = fortrd * f(3) * (f(13) / f(2) - dadm) / f(2)
daddm2 = -fortrd * f(13) / f(2)
dedde = -fortrd * (1$ - 3$ * esq0) * f(3) / f(2)
deddx = dedde * cost0
deddy = dedde * sint0
deddm1 = fortrd * f(5) * eta20 * f(3) / f(2) ^ 2
deddm2 = -fortrd * f(5) * eta20 / f(2)
FOR i% = 1 TO 9
    elemu$(i%) = elem$(i%)
NEXT
'secondcase:
CASE ELSE
tm = time$ / a(17)      'Convert minutes to Hergs
h1 = tm - f(9)
'*** compute position as a function of time (in call line)
osc(9) = h1 * (f(2) + h1 * (f(3) + h1 * f(4)))
osc(3) = dmod(f(1) + osc(9), twopi)
tt = f(5) + f(16) * h1
IF tt < 0$ THEN tt = 0$
IF tt > .99999$ THEN tt = .99999
osc(4) = tt
esq = osc(4) ^ 2
eta2 = 1$ - esq
eta = SQR(eta2)
CALL kepler(osc(3), osc(4), c3)
osc(1) = COS(c3)
c1 = 1$ - osc(4) * osc(1)
osc(1) = (osc(1) - osc(4)) / c1

```

```

osc(2) = eta * SIN(c3) / c1

ts4 = 1# + osc(4) * osc(1)
cf3 = 0#
IF kz% <> 0 THEN cf3 = a(12)
tt = f(13) + f(14) * h1
IF tt < 1# THEN tt = 1#
osc(8) = tt
osc(7) = f(8)
fsi = f(15)
osc(6) = dmod(f(7) + f(12) * osc(9), twopi)
osc(5) = dmod(f(6) + f(11) * osc(9), twopi)

r = osc(8) * eta2 / ts4
osc(6) = osc(6) - cf3 * (tm - a(1))
w1 = COS(osc(5))
w2 = SIN(osc(5))
w7 = COS(osc(6))
w8 = SIN(osc(6))
agda = 0#
agde = 0#
agdi = 0#
agdl = 0#
agdg = 0#
agdh = 0#

w3 = w1 * w1 - w2 * w2
w4 = 2# * w1 * w2
w5 = w3 * w1 - w4 * w2
w6 = w4 * w1 + w3 * w2
w9 = osc(1) * osc(1) - osc(2) * osc(2)
w10 = 2# * osc(1) * osc(2)
w11 = w3 * osc(1) - w4 * osc(2)
w12 = w4 * osc(1) + w3 * osc(2)
w13 = w3 * w9 - w4 * w10
w14 = w4 * w9 + w3 * w10
w15 = w13 * osc(1) - w14 * osc(2)
w16 = w14 * osc(1) + w13 * osc(2)
w17 = atan2(osc(2), osc(1)) + osc(4) * osc(2) - dmod(osc(3), twopi)
w18 = COS(osc(3))
w19 = SIN(osc(3))
w20 = osc(1) * (3# + osc(4) * osc(1) * (3# + osc(4) * osc(1)))
w21 = 3# * w14 + 3# * osc(4) * w12 + osc(4) * w16
w22 = ts4 * (1# + ts4) / eta20
osc(8) = osc(8) * (1# + fsi1 / eta20 * (t12 * (ts4 ^ 3 - t6) -
+ 3# * t15 * ts4 ^ 3 * w13)) + agda
de = vle1 * w3 + vle2 * w2 + vle3 * w6
di = sio2 + .5# * cio2 * (.5# * fs11 * f(8) * f(15) * (3# * w13 + 3# * f(5) -
* w11 + f(5) * w15) - de * f(5) * f(8) / f(15) / eta20) + .5# * cio2 * agdi
de = osc(4) + .5# * fs11 * (t12 * (w20 + f(5) * t8) + 3# * t15 * (w20 + f(5)) -
* w13 - eta20 * t15 * (3# * w11 + w15)) + de + agde
dh = .5# / cio2 * (-.5# * fs11 * f(8) * f(15) * (6# * w17 - w21) + vlhii * w4 -
+ vlh2i * w1 + vlh3i * w5) + .5# * agdh / cio2
d1 = -.25# * t6 * fs11 * (2# * t12 * (w22 + 1#) * osc(2) + 3# * t15 * ((1# -
- w22) * w12 + (.333333333# * w22) * w16))
os = osc(3) + osc(5) + osc(6) - dl * f(5) * (t8 - 1#) / t6 - .25# * fs11 * (6# -
* w17 * (t1 + 2# * f(8)) - w21 * (t1 + 2# * 2# * f(8))) -
+ vls1 * w4 + vls2 * w1 + vls3 * w5 + agdg
dl = dl + eta0 * (vle1 * w4 - vle2 * w1 - vle3 * w5) + agdl
esq = de * de + dl * dl
osc(4) = SQR(esq)
osc(3) = atan2(de * w19 + dl * w18, de * w18 - dl * w19)
osc(7) = 1# - 2# * (di * di + dh * dh)
fsi = SQR(1# - osc(7) * osc(7))
osc(6) = atan2(di * w8 + dh * w7, di * w7 - dh * w8)
osc(5) = os - osc(3) - osc(6)

```

```

eta2 = 1# - esq
eta = SQR(eta2)

CALL kepler(osc(3), osc(4), c3)
osc(1) = COS(c3)
c1 = 1# - osc(4) * osc(1)
osc(1) = (osc(1) - osc(4)) / c1
osc(2) = eta * SIN(c3) / c1

ts4 = 1# + osc(4) * osc(1)
r = osc(8) * eta2 / ts4
w7 = COS(osc(6))
w8 = SIN(osc(6))
w1 = COS(osc(5))
w2 = SIN(osc(5))
ubs = w2 * osc(1) + w1 * osc(2)
ubc = w1 * osc(1) - w2 * osc(2)
u(1) = w7 * ubc - w8 * ubs * osc(7)
u(2) = w8 * ubc + w7 * ubs * osc(7)
u(3) = ubs * fsi
v(1) = -w7 * ubs - w8 * ubc * osc(7)
v(2) = -w8 * ubs + w7 * ubc * osc(7)
v(3) = ubc * fsi
w(1) = w8 * fsi
w(2) = -w7 * fsi
w(3) = osc(7)
ts2 = eta * SQR(osc(8))
FOR i% = 1 TO 3
    vel(i%) = (osc(4) * osc(2) * u(i%) + ts4 * v(i%)) / ts2
NEXT
vel(1) = vel(1) + cf3 * r * u(2)
vel(2) = vel(2) - cf3 * r * u(1)

' Patch for PPT2 to SATSTA output
elemu(1) = osc(8)'semimajor axis
elemu(2) = osc(4)'eccentricity
'elemu(3)=           'time of last perigee passage
elemu(4) = acos(osc(7))'inclination
elemu(5) = osc(6)'node
elemu(6) = osc(5)'arg of perigee
elemu(8) = osc(3)'mean anomaly

ma$ = elemu(8)
CALL kepler(ma$, elemu(2), ea$)
    s0$ = SIN(ea$)
    c0$ = COS(ea$)
'elemu$(8) = ma$
elemu$(3) = time$ - ma$ / elemu$(7)

' in-plane position and velocity
ecc2$ = 1# - elemu$(2) ^ 2
sat.x$(1) = elemu$(1) * (c0$ - elemu$(2))
sat.x$(2) = elemu$(1) * SQR(ecc2$) * s0$
sat.x$(3) = 0$

r$ = elemu$(1) * (1# - elemu$(2) * c0$)
ed$ = elemu$(1) * elemu$(7) / r$
sat.xd$(1) = -elemu$(1) * ed$ * s0$
sat.xd$(2) = elemu$(1) * ed$ * SQR(ecc2$) * c0$
sat.xd$(3) = 0$

' equatorial rectangular coordinates
CALL euler(elemu(6), elemu(5), elemu(4), sat.x(), sat.xd())

END SELECT
END SUB

```

REFERENCES

1. Private communication, Mr. Fred Lipp, Code 20, Naval Space Surveillance Center, Dahlgren, VA 22448.
2. Brouwer, D., "Solution of the Problem of Artificial Satellite Theory without Drag," *The Astronomical Journal*, Vol. 64, No. 1274, Nov. 1959, pp. 378-397.
3. Escobal, P. R., *Methods of Orbit Determination*, John Wiley & Sons, Inc., New York, 1965.
4. Grissom, W., & Guy, R., "Satellite Tracking (SATRAK) Operator's Guide," Version 2.6, Report MG87-SATRAK-HV-6-0001, September 1987, Headquarters Air Force Space Command/DOA, Peterson AFB, CO 80914-5001.
5. Hoots, F. R. and Roehrich, R. L., "Models for Propagation of NORAD Element Sets," Project Spacetrack Report No. 3, December 1980, Aerospace Defense Command, United States Air Force, Peterson AFB, CO.
6. Mikkola, S., "A Cubic Approximation for Kepler's Equation," *Celestial Mechanics*, Vol. 40, Nos. 3-4, 1987, pp. 329-334.
7. Shudde, R. H., "An Analysis of the Satellite Kinematics Model in the Naval Warfare Gaming System," Technical Report NPS55-83-022, September 1983, Naval Postgraduate School, Monterey, CA 93943-5000.
8. Shudde, R. H., "Some Tactical Algorithms for Spherical Geometry," Technical Report NPS55-86-008, March 1986, Naval Postgraduate School, Monterey, CA 93943-5000.
9. Shudde, R. H., "Some Navigation and Almanac Algorithms," Technical Report NPS55-85-023, September 1985, Naval Postgraduate School, Monterey, CA 93943-5000.
10. *The Astronomical Almanac for the Year 1984*, U.S. Government Printing Office, Washington, D.C., 1983.

Intentionally blank.

DISTRIBUTION LIST

	NO. OF COPIES
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5000	2
Office of Research Administration Code 012A Naval Postgraduate School Monterey, CA 93943-5000	1
Library, Code 55 Naval Postgraduate School Monterey, CA 93943-5000	1
Chief of Naval Operations Code OP-953 Washington, D.C. 20350	1
Commander Naval Electronic Systems Command PME 120 Washington, DC 20362	3
Center for Wargaming Naval War College Newport, RI 02840 Attn: Mr. Hap Miller	3
CO Naval Space Surveillance Center Dahlgren, VA 22448 Attn: Mr. Fred Lipp, Code 20	3
Prof. R. N. Forrest, Code 55Fo Naval Postgraduate School Monterey, CA 93943-5000	1
Prof. R. H. Shudde, Code 55Su Naval Postgraduate School Monterey, CA 93943-5000	20